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RESEARCH ARTICLE

Coarse matching with incomplete information

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Abstract We study two-sided markets with heterogeneous, privately informed agents who gain from being matched with better partners from the other side. Our main results quantify the relative attractiveness of a coarse matching scheme consisting of two classes of agents on each side, in terms of matching surplus (output), an intermediary's revenue, and the agents' welfare (defined as the total surplus minus payments to the intermediary). Following Chao and Wilson (Am Econ Rev 77: 899–916, 1987) and McAfee (Econometrica 70:2025–2034, 2002), our philosophy is that, if the worst-case scenario under coarse matching is not too bad relative to what is achievable by more complex, finer schemes, a coarse matching scheme will turn out to be preferable once the various transaction costs associated with fine schemes are taken into account. Similarly, coarse matching schemes can be significantly better than random matching, while still requiring only a minimal amount of information.

Keywords Coarse matching · Incomplete information

JEL Classification C78 · D42 · D82

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1 Introduction

Achieving allocative efficiency when faced with a large diversity of alternatives and preferences requires complex price and allocation schedules. The studies by Chao and Wilson (1987), Wilson (1989) and McAfee (2002) have, however, uncovered instances in which rather simple schemes suffice to obtain most of the efficiency gains from the optimal schemes. Chao and Wilson (1987) and Wilson (1989) consider the case of a monopolist seller faced with a continuum of customers with different valuations for quality, and with a continuum of feasible qualities. They show that the efficiency loss due to the usage of a limited number of quality classes converges to zero at a rate proportional to $1/n^2$, as n, the number of classes, tends to infinity. This elegant result is, however, not very informative about the relative performance of schemes involving a small number of classes. McAfee (2002) addresses this crucial issue in a matching model with a continuum of types on both sides. In his model, efficiency requires assortative matching, pairing the best agent from one side with the best agent from the other side, the second-best with the second-best, and so forth (see also Becker 1973). This matching is contrasted with a scheme involving only two classes: "high" types pay a common price for being randomly matched within the high class, while "low" types get randomly matched within the low class. McAfee's remarkable result is that, for a broad class of distribution of types, the two-class scheme achieves *at least* as much surplus as the average of efficient assortative matching and completely random matching. In other words, the surplus loss associated with this very coarse scheme may be rather modest.

The purpose of this paper is to extend the idea of McAfee (2002) to environments characterized by incomplete information about the agents' types. If types are private information, intermediated exchange consisting of price and matching schedules may induce agents to reveal their private information or parts of it, allowing the implementation of allocations that satisfy some optimality criterion with respect to the underlying preferences. This raises the following issues: even if the coarse scheme performs relatively well in terms of total surplus, it does not follow that it is attractive to a designer who seeks to maximize revenue. Furthermore, governments, for example, may be concerned about the impact of such schemes on the agents' net welfare (defined by the total surplus minus payments to the intermediary). Coarse schemes, involving fixed fees and few matching classes, are indeed often observed in many real-world matching markets.[1] Thus, it seems important to assess the attractiveness of such schemes not only in terms of total surplus, but also in terms of the revenue obtainable by an intermediary, as well as the agents' net welfare. This is the task of the present paper.

In a nutshell, our paper's philosophy is that, if the worst-case scenario is not too bad in terms of total surplus, revenue, and/or the agents' welfare relative to what is achievable by complex, finer schemes, simple coarse matching scheme are prefera-

[1] Examples include employment and recruiting agencies, dating and marriage matchmakers, technology and business brokers. Also, multi-product firms act as intermediaries by matching customers with heterogeneous tastes to products of different qualities. For other examples, see Spulber (2005) survey of the literature on two-sided markets.

ble once the transaction costs associated with fine schemes are taken into account. Casual empiricism suggests that transaction costs are significant. These costs may take the form of: communication, complexity (or menu), and evaluation costs for the intermediary (who needs more detailed information about the environment in order to implement a fine scheme), and for the agents (who need precise information about their own and others' attributes in order to optimally respond to a fine scheme), or higher production costs for firms offering different qualities. In addition, coarse matching schemes have the advantage that agents reveal only partial information about themselves, thus avoiding, at least to some extent, exploitation in the future. Finally, coarse matching schemes using a minimal amount of information may also generate significant welfare and revenue increases with respect to random matching.

Our analysis builds on McAfee's work. Under incomplete information about agents' types, monetary payments are needed in order to implement incentive compatible allocations, while such payments do not appear in McAfee's analysis. In particular, we decompose total welfare under various matching schemes into the intermediary's revenue and the agents' welfare from matching net of payments to the intermediary, and also assess these components separately.

The main tool in McAfee's complete information analysis is the integral form of the so called Chebyshev's inequality which implies that the covariance of two non-decreasing functions of a random variable is non-negative. A main contribution of this paper is the identification of useful techniques from mathematical statistics that yield more refined, quantitative information about this covariance for (large) non-parametric families of random variables arising from convex/concave transformations of the exponential distribution. Roughly speaking, our results use certain bounds on (co)variances, and on the values distribution functions at certain points in their range. These bounds enable us to obtain lower bounds on the total revenue and on net welfare obtainable in two-class matching schemes. Furthermore, by applying these techniques to the analysis of total surplus, we are also able to expand the range of McAfee's main result, while shedding some light on the intuition behind the additional conditions under which his technical analysis was shown to be valid.

An extension of McAfee's matching model that allows for two-sided incomplete information about the agents' types has been introduced by Damiano and Li (2007). These authors derive conditions under which the intermediary achieves a higher revenue by using the efficient scheme (assortative matching) rather than some other (coarser) scheme, and hence the focus of their analysis is quite different from ours. Our central results quantify, within worst-case scenarios, the loss in total surplus, revenue, and net welfare associated with a very coarse scheme involving only two classes, whereas the relative performance of different schemes is not quantified in Damiano and Li.

Chao and Wilson (1987) and Wilson (1989) consider one-sided settings with privately informed customers.[2] But these authors restrict attention to the efficiency effects

[2] Wilson's analysis is extended to the case of multi-unit demand in Spulber (1992).

of coarse allocation schemes, and therefore only those pricing schemes associated to full information revelation/maximum efficiency are characterized. Rayo (2009) looks at a monopolist selling a menu of conspicuous items whose consumption is used as a signal about the agents' hidden characteristics (e.g., luxury goods). By restricting the variety of signals and forcing some subsets of consumers to pool (which corresponds to coarse matching), the monopolist can sometimes extract additional informational rents.[3] Blumrosen and Feldman (2008) and Blumrosen and Holenstein (2008) study mechanism design with restricted action spaces are also closely related to ours. The results of Blumrosen and Feldman, however, focus on total surplus in a one-sided incomplete information setting, similarly to Chao and Wilson (1987) and Wilson (1989). Comparing the optimal mechanism with unrestricted action spaces to mechanisms that employ only k actions, they find that the latter incurs an efficiency loss of $O(1/k^2)$ in their setting. Furthermore, under certain conditions, they show that allocative efficiency may be achieved in environments with two players and two alternatives. For single-item auctions, Blumrosen and Holenstein compare the optimal auction to posted-price auctions in terms of the seller's revenue. It is shown that posted-price auctions with discriminatory (i.e., personalized) prices can be asymptotically equivalent to the optimal full-revelation auctions as the number of bidders increases.

The present paper is organized as follows: In Sect. 2 we describe the extension of McAfee's matching model to environments with privately informed agents on both sides of the market, as introduced by Damiano and Li (2007). Following McAfee (and most of the related matching literature), we assume that the value of a match is the product of the types of the agents (or the product of the agent's type and the good's quality in one-sided models). We further assume that matched agents share the surplus equally. Our analysis easily extends to other fixed sharing rules, as we illustrate in Sect. 5 for a one-sided model. In Sect. 3, we consider the incentive compatible price schedules that lead to assortative matching and coarse matching (Sects. 3.1, 3.2, respectively). For assortative matching, we also establish a connection between those schedules and the unique stable payoff vector in the core of the market.

Section 4 contains our main results. Section 4.1 lists several definitions and results from mathematical statistics which are needed for our subsequent analysis of the relative performance of coarse matching. In Sect. 4.2, we first reconsider McAfee's surplus analysis. Applying our new tools, we show that the surplus loss from coarse matching is less than one-quarter of the total surplus from assortative matching for the class of distributions considered by McAfee. Furthermore, we are able to extend McAfee's result to other classes of distributions. Our findings suggest that two properties of the distribution functions are required to bound the losses from coarse matching: logconcavity and a low degree of variation.

Section 4.3 analyzes the effects of coarse matching on the intermediary's revenue. We establish a counterpart to McAfee's surplus result, which requires a slightly more restrictive condition. The concavity of the distribution functions ensures that the expected utility of agents in the upper class from being randomly matched with

[3] This is related to the "ironing procedures" used in the mechanism design literature.

agents in the lower class is sufficiently small. As a consequence, agents in the upper class have a relatively high willingness to pay for being matched with partners in the upper class. For the two-class scheme, we also characterize the circumstances under which one side of the market pays more to the intermediary than the other. This question is intensely discussed in the literature on two-sided markets, which tends to focus on random matching among homogenous agents and on network externalities,[4] while our explanation is based on heterogeneity differentials.

In Sect. 4.4 we show that the two-class scheme may perform surprisingly well in terms of the agents' welfare when compared to both assortative and random matching. In particular, we identify conditions under which the agents' welfare under coarse matching exceeds (!) the one under assortative matching.[5]

In Sect. 5 we illustrate how the previous results can be applied to a model of price discrimination where a monopolist produces several levels of quality for a market with heterogeneous customers. Section 6 concludes. The Appendix contains several results about the concepts defined in Sect. 4.1, and most of our proofs.

2 The model

We consider the following matching model with incomplete information (see also Damiano and Li 2007): There are two groups of agents, "men" and "women". Each man is characterized by an attribute x, each woman by an attribute y. Agent i's attribute is private information to i. Attributes are distributed according to distributions F (men) and G (women) over the intervals $[0, \tau_F]$ and $[0, \tau_G]$, $\tau_F, \tau_G \leq \infty$, respectively. The distributions F and G are atomless and have continuous densities, $f > 0$ and $g > 0$, respectively. We assume here that the two groups have the same measure, normalized to be one.

An intermediary who cannot observe the agent's types offers contracts that are characterized by a matching rule ϕ, a set-valued function that maps any $x \in [0, \tau_F]$ to a subset $\phi(x) \subseteq [0, \tau_G]$, and by price schedules, $p_m : [0, \tau_F] \to \mathbb{R}$ for men, and $p_w : [0, \tau_G] \to \mathbb{R}$ for women. If man x and woman y are matched, they generate an output (or matching surplus). To capture production complementarities in the simplest way, we assume that the surplus function takes the form $u(x, y) = xy$. We also assume that surplus is shared according to a fixed rule that does not react to market conditions: specifically, if matched, man x receives αxy and woman y receives $(1-\alpha)xy$, where $\alpha \in [0, 1]$.[6] Thus, the utility of a man with attribute x that is matched to a woman with

[4] See, e.g., Armstrong (2006) and Rochet and Tirole (2006) and the literature cited in these papers.

[5] This is related to a result by Hoppe et al. (2009) in a framework where partners use wasteful signals in order to match. They describe circumstances where agents may be better off under random matching (without any waste) than under assortative matching which needs to be sustained by wasteful signaling.

[6] Below we compare this setting to the benchmark where agents share output in a way that guarantees payoffs in the core of the matching market.

attribute y after paying p_m to the intermediary is given by $\alpha xy - p_m$ (and similarly for women).[7]

For any $A \subseteq [0, \tau_F]$ let $\nu_m(A)$ the measure of men announcing types in A. Similarly, define $\nu_w(A)$ for women. The matching rule ϕ is *feasible* if for any $A \subseteq [0, \tau_F]$, $\nu_m(A) = \nu_w(\phi(A))$. That is, each subset of men is matched to a subset of women of equal measure. As shown by Damiano and Li (2007), a feasible and incentive-compatible matching rule partitions the sets of men and women, respectively, into n corresponding subsets, matches the elements of this partition in a positively assortative way, and then, within each matched partition, matches agents randomly.

In this paper, we restrict attention to three special matching rules. First, we consider the limit case where the number of classes n goes to infinity - this is called *assortative matching*. Second, we consider the case of $n = 2$, i.e., *coarse matching* with two classes. Finally, we also consider the case $n = 1$, which yields completely *random matching*.

Throughout the paper we assume that those agents who are not served by the intermediary will be randomly matched with each other.

3 Matching and incentive compatible price schedules

In this section we derive formulas for total surplus, and for the intermediary's revenue obtained when incentive compatible price schedules are used.

3.1 Assortative matching

Under *assortative matching* each man x is matched with a woman $\psi(x)$, where $\psi : [0, \tau_F] \rightarrow [0, \tau_G]$ is implicitly defined by

$$F(x) = G(\psi(x)) \tag{1}$$

If the matching surplus function $u(x, y)$ is supermodular, assortative matching yields the efficient outcome in terms of total output.[8]

In contrast to Damiano and Li (2007), we will express the incentive compatible price schedules in terms of the stable payoffs in the core of the market. This will provide us with useful bounds on the total revenue for the designer. Let δ and ρ denote the surplus shares obtained by man x and woman $\phi(x)$, respectively. Given a matching

[7] The utility function is a straightforward generalization of the standard one-sided independent private value model considered in the auction literature. From that literature it is well known that results beyond the case of risk neutrality are seldom analytically tractable. An advantage of this formulation (which is also used in most of the related matching literature, e.g., Chao and Wilson 1987; McAfee 2002; Damiano and Li 2007) is that all our results can be stated solely in terms of properties of the distribution functions.

[8] A matching surplus is $u(x, y)$ is supermodular if $u_1 > 0$, $u_2 > 0$, and $u_{12} > 0$ where u_i, $i = 1, 2$ denotes the derivative with respect to the ith coordinate, and u_{12} denotes the mixed derivative. It is not difficult to generalize our results about assortative matching (Proposition 1) to the case of a general supermodular matching surplus functions $u(x, y)$.

rule ϕ, the sharing of the surplus among matched agents is called *stable* if:

$$\forall x, \quad \delta(x) + \rho(\phi(x)) = x\phi(x) \tag{2}$$

$$\forall x, y, \quad \delta(x) + \rho(y) \geq xy \tag{3}$$

As efficiency is easily shown to be a prerequisite for stability, the only feasible rule for which the surplus can be shared in a stable way is assortative matching ψ. Let $\varphi = \psi^{-1}$. It is well-known, and straightforward to show that the unique stable shares are given by:

$$\delta(x) = \int_0^x \psi(z)\,\mathrm{d}z \tag{4}$$

$$\rho(y) = \int_0^y \varphi(z)\,\mathrm{d}z \tag{5}$$

In order to implement the assortative matching rule ψ (given a fixed sharing rule α among the matched partners), the intermediary's price schedules p_m and p_w need to satisfy the following incentive-compatibility constraints:

$$\alpha x\psi(x) - p_m(x) \geq \alpha x\psi\left(\hat{x}\right) - p_m\left(\hat{x}\right) \tag{6}$$

$$(1-\alpha)\,\psi^{-1}(y)\,y - p_w(y) \geq (1-\alpha)\,\psi^{-1}\left(\hat{y}\right)y - p_w\left(\hat{y}\right) \tag{7}$$

for all $x, \hat{x} \in [0, \tau]$, and $y, \hat{y} \in [0, \tau]$, respectively.

The following result establishes the relation between stable shares and incentive-compatible price schedules (see the Appendix for the proof).

Proposition 1 1. *The incentive compatible price schedules for the fixed sharing rule α satisfy*

$$p_m(x) = \alpha\rho(\psi(x)) \tag{8}$$

$$p_w(y) = (1-\alpha)\,\delta(\varphi(y))\,, \tag{9}$$

Consequently, the net utilities of any agents x and y are $\alpha\delta(x)$ and $(1-\alpha)\rho(y)$, respectively.

2. *The intermediary's revenue from each matched pair satisfies:*

$$\min(\alpha, 1-\alpha)\,x\psi(x) \leq p_m(x) + p_w(\psi(x)) \leq \max(\alpha, 1-\alpha)\,x\psi(x)$$

3. *In particular, the revenue from each matched pair is exactly half the matching surplus if $\alpha = 1/2$.*

The above result shows that the incentive compatible price schedules effectively correct for the induced distortion of incentives caused by the fact that the fixed sharing rule α does not respond to outside options: the net utilities of the agents form a stable sharing of the output that is left after payments to the intermediary were made.

Total surplus and the intermediary's revenue from price schedules (8) and (9) are, respectively, given by:

$$U^a = \int_0^{\tau_F} x\psi(x)\,\mathrm{d}F(x) \tag{10}$$

$$R_\alpha^a = \alpha \int_0^{\tau_F} \psi(x)\left[x - \frac{1-F(x)}{f(x)}\right]\mathrm{d}F(x) + (1-\alpha)\int_0^{\tau_F} x\left[\psi(x) - \psi'(x)\frac{1-F(x)}{f(x)}\right]\mathrm{d}F(x) \tag{11}$$

Until the application of Sect. 5, we will henceforth assume that surplus is shared equally, i.e., $\alpha = 1/2$, and we will therefore omit this index.

3.2 Coarse matching

We now turn our attention to a matching scheme that involves only two categories on each side of the market. Under coarse matching, the intermediary sets only two prices: p_m^c, p_w^c. Men that are willing (not willing) to pay p_m^c are randomly matched with women that are willing (not willing) to pay p_w^c. Our setting thus captures situations where agents who are not willing to pay still have the possibility to match randomly with each other. The case where excluded agents remain unmatched is discussed below in the context of a multiproduct monopolist (see Sect. 5).

Denote by $\hat{x}(\hat{y})$ the lowest type of men (women) who is willing to pay $p_m^c(p_w^c)$. By the assumptions on match surplus and utility functions, these types are well defined. Such a pricing scheme is incentive compatible if and only if the following equations are satisfied:[9]

$$\alpha\int_0^{\hat{y}} \frac{\hat{x}y}{G(\hat{y})}\mathrm{d}G(y) = \alpha\int_{\hat{y}}^{\tau_G} \frac{\hat{x}y}{1-G(\hat{y})}\mathrm{d}G(y) - p_m^c \tag{12}$$

$$(1-\alpha)\int_0^{\hat{x}} \frac{x\hat{y}}{F(\hat{x})}\mathrm{d}F(x) = (1-\alpha)\int_{\hat{x}}^{\tau_F} \frac{x\hat{y}}{1-F(\hat{x})}\mathrm{d}F(x) - p_w^c \tag{13}$$

$$\hat{y} = \psi(\hat{x}) \tag{14}$$

9 See also Damiano and Li (2007).

The prices p_m^c, p_w^c are such that the cutoff types $\hat{x}$ and $\psi(\hat{x})$ are indifferent between being randomly matched in the high class (while paying the price) and being randomly matched in the low class (for free). Note also that $\hat{y}$ needs to be $\hat{x}$'s partner in assortative matching in order to ensure feasibility.

While Damiano and Li (2007) derive conditions for assortative matching to be optimal in terms of revenue, the present paper aims to assess the circumstances under which the two-class scheme performs relatively well. For this, we seek to find *lower bounds* on the total surplus, revenue and net welfare obtainable through the two-class scheme. Hence, it will suffice to consider only a crude form of coarse matching where the cutoff between the upper and lower classes is determined by the mean of one of the distributions, as in McAfee (2002). Clearly, the lower bounds for either criteria obtained in this paper remain lower bounds if the cutoffs were optimally chosen to maximize surplus, revenue or net welfare, respectively.

Let U^{EX} be the total surplus from coarse matching with two categories where the cutoff point is given by $\hat{x} = EX$, the mean of F. Furthermore, let EX_L be the mean x-type of the low class, and EY_L the mean y-type of the low class when using the cutoffs $\hat{x} = EX$ and $\hat{y} = \psi(EX)$. We have:

$$EX_L = \frac{\int_0^{EX} xf(x)\mathrm{d}x}{F(EX)} = EX - \frac{\int_0^{EX} F(x)\mathrm{d}x}{F(EX)}$$

$$EY_L = \frac{\int_0^{\psi(EX)} yg(y)\mathrm{d}y}{G(\psi(EX))} = \psi(EX) - \frac{\int_0^{\psi(EX)} G(x)\mathrm{d}x}{G(\psi(EX))}$$

Using these definitions, we can write the total surplus as:

$$\begin{aligned} U^{EX} &= \int_0^{EX}\int_0^{\psi(EX)} \frac{xy}{F(EX)} g(y) f(x)\,\mathrm{d}y\,\mathrm{d}x \\ &\quad + \int_{EX}^{\tau_F}\int_{\psi(EX)}^{\tau_G} \frac{xy}{1-F(EX)} g(y) f(x)\,\mathrm{d}y\,\mathrm{d}x \qquad (15) \\ &= EXEY + \frac{F(EX)}{1-F(EX)}(EX - EX_L)(EY - EY_L) \qquad (16) \end{aligned}$$

Similarly, we denote the respective revenue obtained with cutoff $\hat{x} = EX$ by R^{EX}.

For $\alpha = 1/2$, we obtain:

$$\begin{aligned} R^{EX} &= \frac{1}{2}EX\left[\int_{\psi(EX)}^{\tau_G} y\mathrm{d}G(y) - \frac{1-G(\psi(EX))}{G(\psi(EX))}\int_0^{\psi(EX)} y\mathrm{d}G(y)\right] \\ &\quad + \frac{1}{2}\psi(EX)\left[\int_{EX}^{\tau_F} x\mathrm{d}F(x) - \frac{1-F(EX)}{F(EX)}\int_0^{EX} x\mathrm{d}F(x)\right] \\ &= \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] \qquad (17) \end{aligned}$$

4 The relative performance of coarse matching

This section contains our main results. How attractive is coarse matching relative to assortative and random matching? We establish worst-case scenarios from the point of view of total surplus, the intermediary's revenue, and the agents' welfare, while focusing on the case where output is shared equally among matched partners, i.e., $\alpha = 1/2$. In Sect. 5 we provide a discussion of the case of $\alpha = 1$ where one side receives the whole match surplus.[10]

4.1 Failure rates and covariance

We first introduce several definitions and results that will be used in our analysis below:

Definition 1 (1) A distribution function F is said to have an increasing failure rate—IFR (decreasing failure rate—DFR) if $f(t)/[1 - F(t)]$ is increasing (decreasing) on $[0, \tau_F)$, $\tau_F \leq \infty$.[11]

(2) A distribution function F is said to have an increasing reversed failure rate–$IRFR$ (decreasing reversed failure rate—$DRFR$) if $f(t)/F(t)$ is increasing (decreasing) on $[0, \tau_F)$, $\tau_F \leq \infty$.

(3) Let X, Y be non-negative random variables on $[0, \tau_F]$ and $[0, \tau_G]$, $\tau_F, \tau_G \leq \infty$, respectively. The coefficient of covariation of X and Y is given by[12]

$$CCV\ (X, Y)) \equiv \sqrt{Cov(X, Y)/EXEY}$$

In particular, note that $E(XY) = (1 + CCV^2(X, Y))EXEY$ for any two non-negative random variables.

Lemmas 3 and 4 in the Appendix establish various relations among the above defined concepts and exhibit several bounds on the values of distributions with the above properties. The relations among several distribution classes are illustrated in Fig. 1. Two properties of the distribution functions will have important implications

[10] Note that total surplus is not affected by the value of α. The proof of Lemma 2 in Sect. 5 shows how the intermediary's revenue changes when the value of α varies between 0 and 1. Changes in the agents' welfare can then be directly inferred.

[11] For example, the exponential, uniform, normal, power (for $\alpha \geq 1$), Weibull (for $\alpha \geq 1$), gamma (for $\alpha \geq 1$) distributions are *IFR*. The exponential, Weibull (for $0 < \alpha \leq 1$), gamma (for $0 < \alpha \leq 1$) are *DFR*. See Barlow and Proschan (1975) who use the terminology in the context of reliability theory. Other authors refer to "hazard rates".

[12] By the integral form of Chebychevs's inequality (see Theorem 236 in Hardy et al. 1934), $\text{Cov}(X, h(X)) \geq 0$ for any increasing function h. Hence the coefficient of covariation is well defined for any non-negative random variable X, and for any increasing function h. The coefficient of covariation reduces to the more common coefficient of variation $CV(X) \equiv \sqrt{\text{Var}(X)}/EX$ when $X = Y$. This is a dimensionless measure of variability relative to the mean.

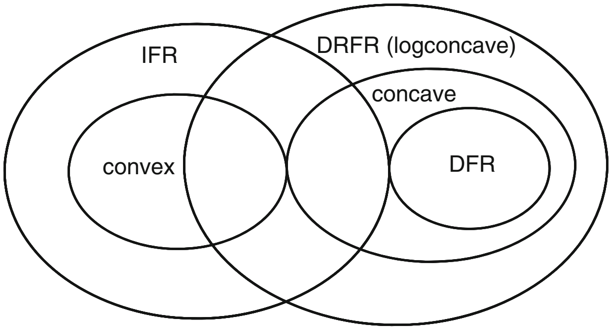

Fig. 1 Distribution classes

for our analysis: First, distributions that fall into the class of logconcave ($DRFR$) or concave distributions on the right side of Fig. 1 can be characterized by sharp upper bounds on the value of the survivor function $1 - F(x)$ at certain points of the distribution (cf. Sengupta and Nanda 1999). Second, distributions that fall into the IFR class on the left side of Fig. 1 have a coefficient of covariation less than 1 (Lemma 3). That is, IFR distributions are characterized by a relatively low degree of variability.[13] An intuitive discussion of the importance of these characteristics for the performance of the different matching schemes will be provided below.

Using the results of Lemmas 3 and 4 (Appendix), we obtain the following lemma (whose proof is in the Appendix), which provides the main working horse for our analysis.

Lemma 1 1. $EX - EX_L \leq (\geq) \frac{1}{2}EX$ *if* F *is convex (concave), and* $\psi(EX) - EY_L \leq (\geq) \frac{1}{2}\psi(EX)$ *if* G *is convex (concave).*
2. $EX - EX_L \leq (\geq) \frac{EXe^{-1}}{F(EX)}$ *if* F *is* IFR (DFR).
3. $EY - EY_L \leq (\geq) \frac{1}{2}EY$ *if* G *is convex (concave) and* ψ *is concave (convex).*
4. $EY - EY_L \geq (\leq) EX - EX_L$ *if* ψ *is convex (concave) and if* $EX \geq (\leq) EY$.

4.2 Total surplus: revisiting McAfee (2002)

In Chao and Wilson (1987) and Wilson (1989) it is shown that the total surplus loss due to the usage of coarse matching with n categories is of order $1/n^2$, that is, $U(n) \geq U^a - O(1/n^2)$, where $U(n)$ denotes the maximal total surplus from coarse matching with n classes. Of course, the order of magnitude in their analyses may still mean that the surplus from coarse matching with only a few categories is small relatively to that in assortative matching. The following elegant result is due to McAfee (2002):

[13] Both, IFR and DFR, are obviously plausible distribution when the random variable is the attribute of an agent (e.g., skill, human capital, health, income), as in our model. Singh and Maddala (1976), for example, take the DFR property as a decisive characteristic of income distributions of U.S. families, while Salem and Mount (1974) advocated the use of an IFR distribution.

Proposition 2 (McAfee 2002) *Let the distributions F and G be both $DRFR$ and IFR. Then*

$$U^{EX} \geq \frac{U^a + U^r}{2} \tag{18}$$

where U^r denotes the total surplus from random matching.

Thus, for a broad class of distributions, the scheme involving only two classes achieves at least as much as the average of assortative and random matching.

We now use Lemma 3(9) (Appendix) in order to establish a tighter, explicit link between the surplus in assortative matching and the surplus in the two-class scheme for the class of distributions considered by McAfee (see the Appendix for the proof):

Corollary 1 *Under the conditions of Proposition 2, we have*

$$U^{EX} \geq \frac{3}{4} U^a. \tag{19}$$

Furthermore, by applying the techniques presented in Sect. 4.1, we are able to provide new insights for the class of DFR distribution functions about which McAfee's result is silent. We first derive lower bounds on the total surplus from two-class coarse matching, expressed as a mark-up on the random output:[14]

Proposition 3 (1) *Let F be DFR, and let ψ be convex. Then*

$$U^{EX} \geq \frac{3}{2} U^r \tag{20}$$

(2) *Let F be DFR, let ψ be convex, and assume that $EX \geq EY$. Then*

$$U^{EX} \geq \frac{e}{e-1} U^r \simeq 1.582 U^r \tag{21}$$

(3) *Let ψ be concave, and switch the role of F and G. Then the above result holds for U^{EY}.*

Proof (1) Since F is DFR and ψ is convex, G must be DFR and hence concave. Thus, since ψ is convex, we know by Lemma 1 (3) that

$$EY - EY_L \geq \frac{1}{2} EY$$

Furthermore, since F is DFR, we have by Lemma 1(2) that

$$EX - EX_L \geq \frac{EX}{eF(EX)}$$

[14] Since $U^r = U^a/[1 + CCV^2(X, \psi(X)]$, this bound can be easily translated into a bound involving the coefficient of covariation and the surplus in assortative matching U^a.

Combining the above insights, we get:

$$\begin{aligned} U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX - EX_L)(EY - EY_L) \\ &\geq EXEY + \frac{F(EX)}{(1-F(EX))}\frac{EX}{eF(EX)}\frac{EY}{2} \\ &= EXEY + \frac{EXEY}{2e(1-F(EX))} \\ &\geq EXEY + \frac{eEXEY}{2e} = \frac{3}{2}EXEY = \frac{3}{2}U^r \end{aligned}$$

(2) By Lemma 1(4) we know that $EY - EY_L \geq EX - EX_L$. Thus, together with Lemma 1(2), we obtain the following chain of inequalities:

$$\begin{aligned} U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX - EX_L)(EY - EY_L) \\ &\geq EXEY + \frac{F(EX)}{1-F(EX)}\left(\frac{EX}{eF(EX)}\right)^2 \\ &= EXEY + \frac{EXEY}{e^2F(EX)(1-F(EX)} \\ &\geq EXEY + \frac{e^2EXEY}{(e-1)e^2} = \frac{e}{e-1}U^r \end{aligned}$$

where the last inequality follows by Lemma 4(7).

(3) This is obvious by the above calculations. □

It is instructive to compare the above result with McAfee's for those distributions where both results apply. Consider $F = G = 1 - e^{-t}$ which is IFR and DFR. In this case, $CCV(X, X) = 1 \Leftrightarrow U^a = 2U^r$, and $U^{EX} = \frac{e}{e-1}U^r$. Thus, our estimate in Part 2 is tight. Note also that $\frac{e}{e-1}U^r = \frac{1}{2}U^r + \frac{2e-1}{2e-2}U^r = \frac{1}{2}U^r + \frac{2e-1}{4e-4}U^a > \frac{1}{2}U^r + \frac{1}{2}U^a$. Thus, by continuity, we obtain a better estimate than McAfee's for IFR distributions that are not too convex with respect to the exponential.

Proposition 3 is now used to extend McAfee's result beyond a subclass of IFR distributions (see the Appendix for the proof).

Proposition 4 *Let F be DFR, ψ be convex, $1 \leq CCV^2(X, \psi(X)) \leq \frac{2}{e-1}$, and $EX \geq EY$.*[15] *Then*

$$U^{EX} \geq \frac{U^a + U^r}{2}$$

Let ψ be concave, and switch the role of F and G. Then the above result holds for U^{EY}.

[15] Alternatively, one could assume that F is DFR, ψ is convex, $1 \leq CV^2(X) \leq \frac{2}{e-1}$, and $EX = EY$.

The following example illustrates the result:

Example 1 Assume that F and G are Weibull: $F(x) = 1 - e^{-x^{\frac{19}{20}}}$, $G(y) = 1 - e^{-y^{\frac{19}{20}}}$ on $[0, \infty)$. Thus, F and G are not IFR, but the conditions of Proposition 4 are satisfied. In fact, $U^{EX} \simeq 1.712$, $[U^a + U^r]/2 \simeq 1.628$, and thus $U^{EX} > [U^a + U^r]/2$.

To understand the intuition behind the results it is helpful to consider Fig. 1 again. First, recall that the property of logconcavity ($DRFR$) provides an upper bound on the survivor function of the distribution at the mean (Sengupta and Nanda 1999). This implies an upper bound on the mass of agents with types above the mean. Since the matching surplus from assortatively matched high types is larger than that from low types, high types contribute more to total surplus than low types. But if the mass of the high-type agents is relatively small, the total surplus loss from matching them randomly within one class instead of assortatively is also relatively small. This tends to make coarse matching attractive.

Second, note that the ratio between the surplus under assortative matching and random matching is determined by the degree of variability of the distributions, i.e. $U^a/U^r = 1 + CCV^2(X, \psi(X))$. Intuitively, as the coefficient of covariation approaches zero, the additional surplus from matching agents assortatively instead of randomly also goes to zero. Therefore, for the two-class scheme to perform sufficiently well, the property of logconcavity needs to be combined with specific bounds on the coefficient of variation. IFR distributions, for example, have a coefficient of variation less than 1. Roughly speaking, if the distribution is IFR, then properly normalized differences between expected values of attributes in successive quantiles are decreasing. This implies a sufficiently low upper bound on the surplus gains from matching agents assortatively instead of randomly in order to establish the results stated in Proposition 2 and Corollary 1. By contrast, DFR distributions are logconcave, but have no upper bound on the coefficient of covariation. Therefore the property of DFR needs to be combined with specific bounds on the coefficient of covariation in order to provide the desired result stated in Proposition 4.

To conclude, the application of the tools identified in Sect. 4.1. to McAfee's matching model delivers the following insight: the efficiency loss associated with a very coarse scheme can be bounded when the mass of agents above the mean is not too big, and when the degree of variability in the population is not too high. As the following analysis will demonstrate, these properties of the distribution functions have similar effects also when other goals are considered.

4.3 The intermediary's revenue

We now turn to the analysis of the relative performance of the two-class scheme in terms of the total revenue obtainable for the intermediary. Taking into account the fact that a larger number of classes will usually be associated with higher transaction costs (or production costs—see the application in Sect. 5), the two-class scheme may be superior to infinitely many classes if the simpler scheme yields a sufficiently high fraction of the revenue achievable from the more complex one. The crucial question of how well the two-class scheme fares against assortative matching in terms of revenue is addressed below.

The methods used in McAfee (2002), in particular Chebyshev's inequality, are not applicable here, and our analysis relies on the results from mathematical statistics identified in Sect. 4.1. Applying these tools, we are able to prove the following:

Proposition 5 *If either (a) F and G are both concave and IFR, or (b) F and G are both DFR, ψ is concave, and $1 \leq CCV^2(X, \psi(X)) \leq \frac{2}{e-1}$, then*

$$R^{EX} \geq \frac{1}{2} R^a \tag{22}$$

Proof (a) By Lemma 1(1), we know that

$$\begin{aligned} EX - EX_L &\geq \frac{1}{2} EX, \text{ and} \\ EY_L &\leq \frac{1}{2}\psi(EX), \text{ and hence } EY - EY_L \geq EY - \frac{1}{2}\psi(EX) \end{aligned}$$

This yields the following inequality chain:

$$\begin{aligned} R^{EX} &= \frac{1}{2}\left[EX(EY - EY_L) + \psi(EX)(EX - EX_L)\right] \\ &\geq \frac{1}{2}\left[EX(EY - \frac{1}{2}\psi(EX)) + \psi(EX)\frac{1}{2}EX\right] \\ &= \frac{1}{2}\left[EXEY - \frac{1}{2}EX\psi(EX) + \frac{1}{2}\psi(EX)\,EX\right] \\ &= \frac{1}{2}U^r = \frac{1}{2}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\ &= \frac{1}{1 + CCV^2(X, \psi(X))} R^a \end{aligned}$$

By Lemma 3(9) we know that $CCV^2(X, \psi(X)) \leq 1$ if F and G are IFR, which yields the result as stated.

(b) By Lemma 1(2), we know that

$$EX - EX_L \geq \frac{EXe^{-1}}{F(EX)}$$

Furthermore, by Lemma 4(3), we have $F(EX) \geq 1 - e^{-1}$ if F is DFR.

Using the fact that G is concave, we obtain the following inequality chain:

$$
\begin{aligned}
R^{EX} &= \frac{1}{2}\left[EX(EY - EY_L) + \psi(EX)(EX - EX_L)\right] \\
&\geq \frac{1}{2}\left[EX(EY - \frac{1}{2}\psi(EX)) + \psi(EX)\frac{1}{2}EX + \psi(EX)\left(\frac{1}{e-1} - \frac{1}{2}\right)EX\right] \\
&= \frac{1}{2}\left[EXEY + \left(\frac{1}{e-1} - \frac{1}{2}\right)\psi(EX)\,EX\right] \\
&\geq \frac{1}{2}\left[EXEY + \left(\frac{1}{e-1} - \frac{1}{2}\right)EYEX\right] \\
&= \frac{1}{4}\frac{e+1}{e-1}U^r = \frac{1}{4}\frac{e+1}{e-1}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\
&= \frac{1}{2}\frac{e+1}{e-1}\frac{1}{1 + CCV^2(X, \psi(X))}R^a
\end{aligned}
$$

where the second inequality is due to Jensen's inequality for concave ψ. For $1 \leq CCV^2(X, \psi(X)) \leq \frac{2}{e-1}$ we obtain the results as stated. □

Proposition 5 identifies conditions under which the revenue from two-class coarse matching is *at least* half the revenue from assortative matching. The proposition thus constitutes the analog of McAfee's surplus result in terms of revenue i.e., $R^{EX} \geq \frac{1}{2}R^a \Leftrightarrow R^{EX} \geq [R^a + R^r]/2$, since the revenue from matching agents randomly is zero.

Part (a) of the above result shows that the revenue relation holds for a subclass of the class of distribution functions identified by McAfee (Proposition 2). That is, while the surplus relation requires $DRFR$ (logconcavity), concavity is needed here, which is stronger (cf. Lemma 3 in the Appendix). Examples include the uniform distribution and the exponential distribution.

Example 2 Assume that $F = x^3$, $G = y^3$ on $[0, 1]$. F, G are then IFR and $DRFR$ (but not concave). In fact, $R^{EX} < \frac{1}{2}R^a$.

Furthermore, part (b) reveals that the requirement of IFR can be abandoned if the property of DFR is combined with specific bounds on the coefficient of covariation, similar as in our analysis of total surplus effects.[16]

The main message is, again, that the two general properties highlighted in our previous surplus analysis, (log)concavity and a low degree of variation, play an important role here as well. First, notice that payments are obtained only from agents in the high class. Their willingness to pay for being matched with a random partner from the high class on the other side of the market is determined by the value of the outside option, i.e. being randomly matched with a partner from the low class. Concavity imposes a

[16] While the upper bound on the coefficient of covariation is identical to that derived in Proposition 4, we obtain a different condition on the shape of the assortative matching function ψ here. We suspect that the conditions on ψ are not necessary. However, we have not been able to prove the results without these requirements.

lower upper bound on the mass of agents with types above the mean than logconcavity (Sengupta and Nanda 1999). As a consequence, agents above the mean are willing to pay more for being matched in the high class, leading to a higher revenue for the intermediary. Second, the revenue from assortative matching is tightly linked to the assortative surplus (Proposition 1(3)). Thus, as in the analysis of total surplus, concavity needs to be combined with a property ensuring that this coefficient of covariation is bounded from above (such as IFR). Likewise, the condition of DFR needs to be paired with specific bounds on the coefficient of covariation.

Which side pays more? In practice, price schedules observed in two-sided markets are often uneven, with one side paying more than the other.[17] Explanations of this asymmetry tend to focus on network externalities.[18] A result in Hoppe et al. (2009) can be adapted to show that men's total payment is larger (smaller) than women's total payment when the intermediary uses assortative matching if the matching function $\psi = G^{-1}F$ is convex (concave). In particular, agents are willing to pay more as the other side becomes more heterogeneous (since then the marginal gains in terms of winning a better matching partner are larger at the high end of the type range).

We inquire here whether this finding carries over to coarse matching schemes. Let R_m^{EX} be the total payments obtained from men, and R_w^{EX} the total payments obtained from women under the two-class scheme.

Proposition 6 (1) *Assume that either (a) F is convex and G concave, or (b) F is IFR and G is DFR. Then ψ is convex and $R_m^{EX} \geq R_w^{EX}$.*
(2) *Assume that $EX \geq EY$ and that ψ is convex. Then $R_m^{EX} \geq R_w^{EX}$.*

Proposition 6 resembles the finding for assortative matching, but the intuition is here slightly different. Under coarse matching, the agents' willingness to pay is determined by the outside option of being randomly matched within the low class. If F and G have the same mean, but the distribution of women, G, has a higher variance, the chances for men who take the outside option to end up with a very low type partner are higher than for women. As a consequence, for types in the high class, men's willingness to pay is larger than women's.[19]

4.4 The agents' welfare

Another important criterion for measuring the relative performance of coarse matching is the agents' welfare, defined as total surplus minus the total payment to the intermediary. In markets with private intermediaries, governments may care about the agents' welfare even if they do not care about full efficiency. A priori, it is not clear how the loss in total surplus associated with a coarse matching scheme is shared among the

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17 See, e.g., *The Economist*, "Matchmakers and trustbusters", p. 84, December 10th, 2005.

18 For an analysis of two-sided markets with network externalities, see, for instance, Rochet and Tirole (2003).

19 In addition, there is a size effect (this is similar to assortative matching): if ψ is convex, the mass of men with types above a certain threshold is larger than the mass of women with types above that threshold.

various parties: since both the total surplus and revenue may be larger under assortative matching, the effect on the agents' welfare is, in principle, ambiguous.

Let $W^a = U^a - R^a$ and $W^{EX} = U^{EX} - R^{EX}$ be the agents' welfare under assortative matching and under coarse matching with cutoff EX, respectively, and recall that $W^r = U^r$ under random matching since the intermediary's revenue is then zero. Our comparison of the agents' welfare under these schemes relies again on the techniques of Sect. 4.1.

Proposition 7 (1) *Let F and G be concave and IFR, and let ψ be convex. Then*

$$W^{EX} \geq \frac{3}{4} W^a \tag{23}$$

(2) *Let F and G be $DRFR$ and convex. Then*

$$W^{EX} \geq W^a \tag{24}$$

The proposition clearly indicates that coarse matching performs well in terms of the agents' welfare. The idea of the proof is to combine upper bounds on the revenue from coarse matching with lower bounds on the total surplus.

It is interesting to note that Part 1 suggests that, for distributions that are concave and IFR, the coarse scheme performs relatively well in all three categories, total surplus, revenue, and the agents' welfare. Part 2 of Proposition 7 even identifies a class of distributions for which the agents' welfare exceeds the one obtainable under assortative matching! From our previous analysis, we know that for this class of distributions, coarse matching is relatively strong in terms of surplus (Proposition 2), but tends to be weak in terms of revenue (Proposition 5). Intuitively, the combination of these two features strengthens coarse matching from a net welfare point of view.

Example 3 Assume that F, G are uniform on [0, 1]. Thus, the distributions are convex and $DRFR$. we get $W^{EX} = 3/16$, $W^a = 1/6$, and thus $W^{EX}/W^a = 9/8$.

The next proposition shows that coarse matching, using a minimal amount of information, performs relatively well compared to random matching for the class of DFR distributions.

Proposition 8 *Let F and G be DFR and let ψ be convex. Then*

$$W^{EX} \geq \frac{3}{4} W^r \tag{25}$$

Recall that random matching generates zero revenue. In practice, many intermediaries must, however, respect a budget constraint. Together with the insight for revenue (Proposition 5), the previous result suggest that an intermediary who wishes to maximize the agents' welfare while collecting sufficient revenue to recover its costs of capital (analogous to the Ramsey–Boiteux problem in the theory of public finance) may find a coarse scheme to be quite attractive relative to both assortative and random matching.

Finally, note that in some cases there is a significant tension between the intermediary's and the agents' views about the optimal number of matching classes. The conditions identified in Damiano and Li (2007) for having a maximal revenue from assortative matching are violated, for example, when the distributions are DFR. Thus, in such cases more pooling (e.g. achieved via coarse matching with less classes) should be advantageous for the intermediary, analogously to the ironing procedures used in the mechanism design literature whenever virtual values are not increasing. But, in those cases, pooling is detrimental for the agents since having less classes also decreases total surplus.

5 An application to one-sided incomplete information models

Throughout the above analysis, we assumed that privately informed agents populate both market sides. Here we briefly illustrate how some of our previous insights can be modified for a context where there are privately informed agents only on one side who get matched to observable items (or partners) on the other side.[20] For instance, Wilson (1989) studies a multi-product seller in the electricity industry, seeking to match customers having heterogeneous valuations for power provision to different service qualities that represent service probabilities. In this case, consumers naturally obtain the whole surplus from their purchase minus their payment to the seller. In other words the sharing rule corresponds to $\alpha = 1$, and we have to adjust our previous revenue and welfare results for this case. Total surplus is of course unaffected by the value of α. For $\alpha = 1$, total revenue from assortative matching is given by

$$R_1^a = \int_0^{\tau_F} \psi(x) \left(x - \frac{1 - F(x)}{f(x)} \right) \mathrm{d}F(x) \tag{26}$$

and the total revenue from coarse matching is given by

$$R_1^{EX} = EX(EY - EY_L) \tag{27}$$

Simple relations between revenues for $\alpha = 1$ and $\alpha = 1/2$ are:

Lemma 2 *If the assortative matching function ψ is convex (concave) then $R_1^a \geq (\leq) R_{1/2}^a$. If the assortative matching function ψ is convex (concave) and if $EX \geq (\leq) EY$ then $R_1^{EX} \geq (\leq) R_{1/2}^{EX}$.*

The next two results illustrate how our previous results can be adapted to this framework, and provide a comparison of the two matching rules in terms of revenue and agents' welfare:

Proposition 9 *Let F and G be IFR and concave, and ψ be convex. Then $R_1^{EX} \geq \frac{1}{4} R_1^a$.*

[20] The classical references are Mussa and Rosen (1978), and Maskin and Riley (1984).

Proposition 10 (1) *Let F and G be IFR and concave, and let ψ be convex. Then* $W_1^{EX} \geq \frac{1}{2} W_1^a$.
(2) *Let F and G be convex and $DRFR$, and let ψ be convex. Then* $W_1^{EX} \geq W_1^a$.

In Wilson's model, the distributions of customers' valuations and the feasible distribution of service probabilities (and hence the assortative matching function ψ) are exogenously given.[21] Propositions 9 and 10 can therefore be directly used to identify lower bounds on the revenue and/or the agents' welfare associated with two service classes.

5.1 Price discrimination with quality costs

Here we briefly illustrate how Propositions 9 and 10 can be applied to the more realistic situation in which the quality levels are not exogenously given, but where the monopolist takes production costs into account when determining the available qualities. In contrast to Wilson's model, the assortative matching function will be now endogenously determined.

We denote quality by q. There is a measure one of consumers, each demanding a unit of the good. Consumers are distributed over $[0, 1]$ according to distribution F with $f = F' > 0$. The utility of type v from quality q is vq. The cost of producing y units of quality q is $c(q)y$ where $c(0) = c'(0) = 0$ and where $c'(q) > 0$ and $c''(q) > 0$ for $q > 0$.

Consider first a monopolist who uses standard non-linear pricing. In this case, the monopolist chooses a menu of prices and qualities from which the consumers can pick their preferred combination. We make the standard assumption that the virtual valuation $v - (1 - F(v))/f(v)$ is increasing, which holds, for example, if F is IFR. Denote by $(p(v), q(v))$ the element that is chosen by type v. Using standard arguments, the monopolist's revenue and profit under incentive compatible price schedules and assortative matching are given, respectively, by:

$$R_1^a = \int_0^1 q(v)\left(v - \frac{1 - F(v)}{f(v)}\right) f(v)\,\mathrm{d}v$$

$$\pi_1^a = \int_0^1 \left[q(v)\left(v - \frac{1 - F(v)}{f(v)}\right) - c(q(v))\right] f(v)\,\mathrm{d}v.$$

Let r be such that $r - (1 - F(r))/f(r) = 0$. The function q that maximizes the above profit function is determined by

[21] See McAfee (2002) for a mapping of Wilson's set-up into the matching framework used here (albeit with complete information).

$$q(v) = 0 \quad \text{if } v \le r$$
$$c'(q(v)) = v - \frac{1 - F(v)}{f(v)} \quad \text{if } v \ge r$$

The profit maximizing menu contains a continuum of quality levels. Denote the distribution of quality levels by G, and note that $G(y) = F(q^{-1}(y))$ for $y \in (0, q(1)]$. Thus, the function q plays here the role of the assortative matching function ψ.[22] Note that assumptions on the quality costs will play a similar role as the conditions on G in our two-sided setting.

We first compare the revenue from the optimal menu derived above to the revenue that can be achieved by producing only two quality levels, and charging two prices. Suppose that customers with valuations below $EV = \int_0^1 v\mathrm{d}F(v)$ are matched with the quality level $Q^L = \int_0^{EV} q(z)\mathrm{d}F(z)/F(EV)$ and customers with valuations above this cutoff are matched with the quality level $Q^H = \int_{EV}^1 q(z)\mathrm{d}F(z)/(1 - F(EV)) = \frac{EQ - F(EV)Q^L}{1 - F(EV)}$.

Let p^L and p^H be the prices for low and high quality, respectively. By incentive compatibility these prices satisfy

$$\begin{aligned} p^H &= \frac{EV \int_{EV}^1 q(x)\, dF(x)}{1 - F(EV)} - \frac{EV \int_0^{EV} q(x)\, dF(x)}{F(EV)} + p^L \\ &= EV\left(Q^H - Q^L\right) + p^L \end{aligned}$$

The purpose of our analysis is to find a lower bound on the revenue associated with the two-quality scheme. For such a worst-case scenario, it suffices to consider the case of $p^L = 0$, since this case clearly underestimates the maximal revenue from coarse matching.[23] The revenue from the coarse provision of quality is then given by:

$$R_1^{EV} = EV\left(EQ - Q^L\right) \tag{28}$$

which is analogous to expression (27). Hence, Propositions 9 and 10 can be directly used to obtain revenue and welfare results for specific cost functions, as we illustrate in the following example.

Example 4 Suppose that $c(q) = q^2$ and that v is uniformly distributed over [0, 1]. In this case, $q(v) = 0$ if $v \le \frac{1}{2}$ and $q(v) = \frac{2v-1}{2}$ if $v > \frac{1}{2}$. Thus $G(y) = \frac{1+2y}{2}$ for $y \in [0, \frac{1}{2}]$, which is concave and IFR. Although $G(0) > 0$, we get $CCV^2(X, \psi(X)) \le 1$,

[22] Although $q^{-1}(0)$ is not defined here, q can be approximated by an everywhere strictly increasing function, allowing us to use previous results. In fact, since the monopolist does not get revenue from agents that are not served, the results from the approximation underestimate the ratio between revenue in coarse matching versus revenue in assortative matching, since only a fraction of the agents are assortatively matched with a positive quality.

[23] A price-discriminating monopolist can either set Q^L at zero or increase p^L to a positive value (thus excluding some customers) in order to increase revenue. Note that for maximizing the agents' net welfare, $p^L = 0$.

which is required for an application of Proposition 9. Straightforward algebra yields $Q^H = \frac{1}{2}$, $R_1^a = \frac{1}{12}$, $R_1^{EV} \geq \frac{1}{16}$, and thus $R_1^{EV} \geq \frac{3}{4} R_1^a$. Furthermore, considering total profit (that takes into account the production cost) we get $\pi_1^a = \frac{1}{24}$, $\pi_1^{EV} \geq \frac{1}{32}$, and $\pi_1^{EV} \geq \frac{3}{4}\pi^a$. Moreover, the conditions of Proposition 10(1) are also satisfied, which yields $W^{EV} \geq \frac{1}{2} W^a$.

Note that Proposition 9 deals only with revenue. What can be said about the profit comparison? Since costs are convex, the monopolist saves costs by producing the average quality levels Q^L and Q^H rather than producing the whole range of qualities. This does not immediately translate into a lower bound on profits from coarse provision of quality. Yet, if the cost function is sufficiently convex, the cost savings will be substantial and the bound on profits will match the bound on revenues (see the above example). If various transaction costs of providing different qualities are also present (e.g. cost of re-tooling machines, of marketing, etc.), our analysis suggests that the coarse provision of quality may well be the profit-maximizing choice of the monopolist for a broad range of customer type distributions and cost functions. Moreover, a public agency that is interested in the agents's welfare but has some revenue considerations (for budgetary reasons, etc.) will also have a strong incentive to implement coarse matching.

6 Conclusion

We have studied the performance of very coarse matching schemes and the associated price schedules in a two-sided market with heterogenous agents who are privately informed about their attributes. In a variety of settings we have shown that such schemes are very effective. The type of analysis performed in this paper is not standard in the Economics literature, which tends to concentrate on the zero-one dichotomy between optimality versus suboptimality: degrees of suboptimality are not quantified or compared. Several papers in mechanism design argue that only "simple" mechanisms are realistic.[24] But, a majority of these follow the above dichotomy, by completely specifying special settings where simple mechanisms are *fully* optimal.[25] Instead, we focus on a priori *suboptimal* mechanisms, while identifying settings where such mechanisms are very effective (and thus may become optimal once transaction costs associated with more complex mechanisms are taken into account).[26] The scarcity of "worst-case" studies (or of other quantifications of suboptimality) in the Economics literature should be contrasted to the wealth of papers following precisely this

[24] This is one argument in what is sometimes called the "Wilson doctrine" (see Wilson 1987).

[25] E.g, Myerson (1981) shows that standard one-object auctions with a reserve price are revenue maximizing in the symmetric, independent private value environment. Holmstrom and Milgrom (1987) identify conditions where linear contracts are optimal for the provision of intertemporal incentives in a principal-agent relationship.

[26] Our analysis is similar in spirit to the study by Neeman (2003) about the effectiveness of the English auction in environments where this auction is not revenue maximizing. See also Rogerson (2003) who describes an example where simple contracts achieve a substantial shares of the profit obtainable by full non-linear pricing in a cost-based procurement and regulation framework.

philosophy in the Operations Research/Computer Science literature.[27] We believe that both our understanding of existing trading institutions and our ability to design new effective institutions will profit from more studies in this vein.

Appendix

The next lemma gathers several main relations among the concepts defined in Sect. 4.1. For the less obvious implications, see Barlow and Proschan (1975). We first need the following definition:

Definition 2 A distribution function F is said to be new better than used in expectation—$NBUE$ (new worse than used in expectation—$NWUE$) if $\int_t^\tau (1 - F(x))\mathrm{d}x \leq (\geq) EX(1 - F(t)), t \geq 0$.

Lemma 3 1. *Any IFR distribution is $NBUE$.*
2. *A distribution $F$is IFR if and only if its survivor function $(1 - F)$ is logconcave.*
3. *Any DFR distribution is $NWUE$.*
4. *Any convex distribution is IFR.*
5. *Any DFRdistribution is concave.*
6. *Any concave distribution is $DRFR$.*
7. *A distribution F is $DRFR$if and only if F is logconcave.*
8. $CV^2(X) \leq (\geq) 1$ *if F, the distribution of X, is $NBUE$ $(NWUE)$.*
9. $CCV^2(X, G^{-1}F(X)) \leq (\geq)\ 1$ *if both F and G are $IFR(DFR)$distributions.*

Properties such as mentioned in the above lemma are of interest here since they describe large, non-parametric classes of distribution functions for which various upper or lower bounds on the values of distributions at various points in their respective range hold. Such bounds are exhibited in the next lemma:

Lemma 4 1. $F(EX)(1 - F(EX)) \leq \frac{1}{4}$*for all F.*
2. $F(EX) \leq (\geq) \frac{1}{2}$ *if F is convex (concave).*
3. $F(t) \leq (\geq) 1 - e^{-\frac{t}{EX}}$ *, $t < EX$ $(t \leq EX)$ if F is IFR (DFR).*
4. $\int_0^\tau (1 - F(x))\mathrm{d}x = EX$ *for all F.*
5. $\int_0^t F(x)\mathrm{d}x \leq (\geq) t - EX(1 - e^{-\frac{t}{EX}})$ *if F is $NBUE(NWUE)$.*
6. $\int_0^{EX} F(x))\mathrm{d}x \leq (\geq)\ EXe^{-1}$ *if F is $NBUE(NWUE)$.*
7. $F(EX)(1 - F(EX)) \leq \frac{e-1}{e^2} \approx 0.23254$ *if $F$is DFR.*

Proof (1) This is immediate by observing that on the interval [0, 1] the function $x(1 - x)$ has a maximum at $x = \frac{1}{2}$.
(2) Note that $F(X)$ is a uniformly distributed random variable on the interval [0, 1], and hence $E[F(X)] = \frac{1}{2}$. The claim follows then by Jensen's inequality since $F(EX) \leq (\geq) E[F(X)] = \frac{1}{2}$ if F is convex (concave).

[27] Koutsoupias and Papadimitriou (1999) and Roughgarden and Tardos (2004) are excellent examples since they also combine game-theoretic reasoning. These authors study the "price of anarchy" in network routing games. This is defined as the ratio between the welfare in the worst Nash equilibrium and the welfare in the Pareto-optimal allocation.

(3) The assertions are contained in Theorems 4.4 and 4.7 in Barlow and Proschan (1965).
(4) This is immediate from integration by parts.
(5) This follows by 4) and by rearrangement of terms from the fact that $\int_t^\tau (1 - F(x))\mathrm{d}x \leq (\geq)\ EXe^{-\frac{t}{EX}}$ if F is $NBUE(NWUE)$ (see Barlow and Proschan 1975, page 187).
(6) This is just the instance of (5) for $t = EX$.
(7) By 3) for $t = EX$, we know that $F(EX) \geq 1 - e^{-1}$ for F DFR. Since $1 - e^{-1} \geq \frac{1}{2}$, we obtain that the function $x(1-x)$ is decreasing for $x \geq 1 - e^{-1}$. Thus, $F(EX)(1 - F(EX)) \leq (1 - e^{-1})(e^{-1}) = \frac{e-1}{e^2}$.

□

Proof of Proposition 1 Using the envelope theorem and condition (6), we obtain

$$\alpha x \psi'(x) - \frac{\mathrm{d}p_m(x)}{\mathrm{d}x} = 0.$$

Since the man with the lowest type will be matched for sure with the woman with the lowest type, the willingness to pay of this type is always zero, which yields the boundary condition $p_m(0) = 0$. Hence, we obtain

$$p_m(x) = \int_0^x \alpha z \psi'(z)\,\mathrm{d}z. \tag{29}$$

The incentive-compatible price schedule for women is analogously derived. Letting $\varphi = \psi^{-1}$, we have

$$p_w(y) = \int_0^y (1 - \alpha)\, z \varphi'(z)\,\mathrm{d}z. \tag{30}$$

To derive the net utilities of matched agents x and $\psi(x)$, note that

$$u(x, \psi(x)) = \int_0^x \psi(z)\,\mathrm{d}z + \int_0^x z\psi'(z)\,\mathrm{d}z$$

Thus, the net utilities of agents x and $\psi(x)$ from contracting with the intermediary (and being matched with each other) can be written, respectively, as

$$\alpha x \psi(x) - p_m(x) = \alpha \int_0^x \psi(z)\,\mathrm{d}z = \alpha\delta(x)$$

$$(1 - \alpha)\, x\psi(x) - p_w(\psi(x)) = (1 - \alpha) \int_0^y \varphi(z)\,\mathrm{d}z = (1 - \alpha)\rho(y)$$

where $\delta(x)$, $\rho(y)$ are the stable shares, respectively. □

Proof of Lemma 1 (1) Geometrically, the term $[\int_0^{EX} F(x)\mathrm{d}x]/[EXF(EX)]$ is the ratio among the area below F up to the mean, and the area of the rectangle with sides of EX and $F(EX)$, respectively. Note that any chord to the graph of a continuous convex (concave) function lies entirely above (below) the graph. Thus $\int_0^{EX} F(x)\mathrm{d}x$ covers less (more) than $\frac{1}{2}$ of the area of the rectangle when F is convex (concave). Observe that the equality $EX - EX_L = \frac{1}{2}EX$ is indeed obtained for the uniform distribution (on any interval), which is both convex and concave. By the same geometric argument, we get $\psi(EX) - EY_L = [\int_0^{\psi(EX)} G(x)\mathrm{d}x]/[G(\psi(EX))] \le (\ge)\frac{1}{2}\psi(EX)$ if G is convex (concave).

(2) This follows immediately from Lemma 3(1), (3) and Lemma 4(6).

(3) The result follows from by statement 1 and because $\psi(EX) \ge (\le) E(\psi X) = EY$if ψ is concave (convex) by Jensen's inequality.

(4) Assume first that ψ is convex. If $F = G$, the result is obvious. Thus, assume $F \ne G$. By Theorem 6.2 in Barlow and Proschan (1981) $F(t) \le G(t)$ for $t < EX$. In other words, the unique crossing of F and G (which must exists in this case) cannot occur below $t = EX$. Since $F \ne G$, we obtain $\psi(EX) < E(\psi X) = EY \le EX$. Thus

$$F(t) \le G(t) \quad \text{for } t \le \psi(EX)$$

This yields the following chain:

$$\begin{aligned}
&\frac{\int_0^{\psi(EX)} G(t)\mathrm{d}t}{G(\psi(EX))} \ge \frac{\int_0^{\psi(EX)} F(t)\mathrm{d}t}{F(EX)} \\
&\Leftrightarrow \frac{\int_0^{\psi(EX)} G(t)\mathrm{d}t}{G(\psi(EX))} + \frac{\int_{\psi(EX)}^{EX} F(EX)\mathrm{d}t}{G(\psi(EX))} \ge \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} + \frac{\int_{\psi(EX)}^{EX} F(EX)\mathrm{d}t}{G(\psi(EX))} \\
&\Leftrightarrow \frac{\int_0^{\psi(EX)} G(t)\mathrm{d}t}{G(\psi(EX))} + \frac{\int_{\psi(EX)}^{EX} F(EX)\mathrm{d}t}{G(\psi(EX))} \ge \frac{\int_0^{\psi(EX)} F(t)\mathrm{d}t}{F(EX)} + \frac{\int_{\psi(EX)}^{EX} F(t)\mathrm{d}t}{G(\psi(EX))} \\
&\Leftrightarrow EX - \psi(EX) + \frac{\int_0^{\psi(EX)} G(t)\mathrm{d}t}{G(\psi(EX))} \ge \frac{\int_0^{EX} F(t)\mathrm{d}t}{F(EX)} \\
&\Leftrightarrow EY - \left(\psi(EX) - \frac{\int_0^{\psi(EX)} G(t)\mathrm{d}t}{G(\psi(EX))}\right) \ge \frac{\int_0^{EX} F(t)\mathrm{d}t}{F(EX)} \\
&\Leftrightarrow EY - EY_L \ge EX - EX_L
\end{aligned}$$

The first inequality holds by the above observation and because $G(\psi(EX)) = F(EX)$. The third holds because F is increasing.

If ψ is concave, then ψ^{-1} is convex, and the argument holds with reversed roles. □

Proof of Corollary 1 The result follows because $U^r = U^a/[1 + CCV^2(X, \psi(X)]$ and because $CCV^2(X, \psi(X)) \le 1$ by Lemma 3(9). □

Proof of Proposition 4 Assume that F be DFR, ψ be convex, and that $EX \ge EY$. We need to show that $[U^{EX} - U^r]/[U^a - U^r] \ge \frac{1}{2}$. Note that $U^a - U^r =$

$CCV^2(X, \psi(X))U^r$, and $U^{EX} - U^r \geq \frac{1}{e-1}U^r$ by Proposition 3(2). Solving for $CCV^2(X, \psi(X))$ yields the result as stated. □

Proof of Proposition 6

(1) (a) By the concavity of G, we get that $R_m^{EX} = \frac{1}{2}EX(EY - EY_L) \geq \frac{1}{4}EXEY$. By the convexity of F and ψ, we get $R_w^{EX} = \frac{1}{2}\psi(EX)(EX - EX_L) \leq \frac{1}{2}EY(EX - EX_L) \leq \frac{1}{4}EXEY$. The result follows.
b) Because G is DFR and ψ is convex, using Lemma 4(3), we obtain the following chain:

$$\begin{aligned} EY - EY_L &= EY - \psi(EX) + \frac{\int_0^{\psi(EX)} G(t)\mathrm{d}t}{G(\psi(EX))} \\ &\geq EY - \psi(EX) + \frac{\int_0^{\psi(EX)}(1 - e^{-\frac{t}{EY}})\mathrm{d}t}{(G\psi(EX))} \\ &= EY - \psi(EX) + \frac{\psi(EX) - EY + EYe^{-\frac{\psi(EX)}{EY}}}{G(\psi(EX))} \\ &\geq \frac{(EY - \psi(EX))(G(\psi(EX)) + \psi(EX) - EY + EY(1 - G(\psi(EX)))}{G(\psi(EX)} \\ &= \frac{\psi(EX)(1 - G(\psi(EX))}{G(\psi(EX))} \end{aligned}$$

This yields the following chain:

$$\begin{aligned} R_m^{EX} &= \frac{1}{2}EX(EY - EY_L) \geq \frac{EX\psi(EX)(1 - G(\psi(EX))}{2G(\psi(EX))} \\ &= \frac{EX\psi(EX)(1 - F(EX)}{2F(EX)} \geq \frac{1}{2}\psi(EX)(EX - EX_L) = R_w^{EX} \end{aligned}$$

where the last inequality follows because F is IFR.

(2) From Lemma 1(4) we know that $EY - EY_L \geq EX - EX_L$. Since $EX = EY \geq \psi(EX)$, we obtain $R_m^{EX} = EX(EY - EY_L) \geq \psi(EX)(EX - EX_L) = R_w^{EX}$.

Proof of Proposition 7 To prove the proposition we first need to derive upper bounds on the revenue from coarse matching:

Lemma 5 (1) *Let F be concave (convex), ψ be convex (concave). Then*

$$R^{EX} \leq (\geq)\frac{1}{2}\left(U^{EX} - EX_LEY_L\right) \tag{31}$$

(2) *Let F and G be convex. Then*

$$R^{EX} < \frac{1}{2}U^{EX} \tag{32}$$

Proof For the first part, consider the following chain that holds for ψ convex and F concave (the other direction is analogous):

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX-EX_L)(EY-EY_L)\\
&\geq EXEY + (EX-EX_L)(EY-EY_L)\\
&= 2EXEY - EXEY_L - EYEX_L + EX_LEY_L\\
&= 2\left[\frac{1}{2}(EX(EY-EY_L) + EY(EX-EX_L))\right] + EX_LEY_L\\
&\geq 2\left[\frac{1}{2}(EX(EY-EY_L) + \psi(EX)(EX-EX_L))\right] + EX_LEY_L\\
&= 2R^{EX} + EX_LEY_L
\end{aligned}$$

The first inequality follows from Lemma 4(1) and the second inequality holds since $EY \geq \psi(EX)$ for ψ convex. The last equality uses formula (17).

For the second case where F, G are both convex, we use (17) to obtain:

$$\begin{aligned}
R^{EX} &= \frac{1}{2}[EX(EY-EY_L) + \psi\,(EX)\,(EX-EX_L)]\\
&\leq \frac{1}{2}\left[EX(EY - \frac{1}{2}\psi(EX)) + \psi\,(EX)\,\frac{1}{2}EX\right]\\
&= \frac{1}{2}\left[EXEY - \frac{1}{2}EX\psi(EX) + \frac{1}{2}\psi\,(EX)\,EX\right]\\
&= \frac{1}{2}U^r < \frac{1}{2}U^{EX}
\end{aligned}$$

where the first inequality follows from Lemma 1. □

(1) By Lemma 5(1), we know that $R^{EX} < \frac{1}{2}U^{EX.}$. Hence,

$$\begin{aligned}
W^{EX} &= U^{EX} - R^{EX} > \frac{1}{2}U^{EX.} \geq \frac{1}{4}(U^a + U^r)\\
&\geq \frac{1}{4}\left(U^a + \frac{1}{2}U^a\right) = \frac{3}{8}U^a = \frac{3}{4}W^a
\end{aligned}$$

The first inequality follows from $R^{EX} < \frac{1}{2}U^{EX.}$. The second follows by McAfee's result (recall that any concave distribution is $DRFR$). The last equality follows by Proposition 1.

(2) We know from the proof of Lemma 5 that $R^{EX} \leq \frac{1}{2}U^r$ for F and G convex. This gives:

$$W^{EX} = U^{EX} - R^{EX} \geq \frac{1}{2}\left(U^a + U^r\right) - \frac{1}{2}U^r = \frac{1}{2}U^a = W^a$$

where the first inequality follows from $R^{EX} \leq \frac{1}{2}U^r$ together with McAfee's result (recall that convex distributions are IFR), and the last equality follows by Proposition 1. □

Proof of Proposition 8 We have the chain:

$$W^{EX} = U^{EX} - R^{EX} \geq U^{EX} - \frac{1}{2}\left(U^{EX} - EX_L EY_L\right) = \frac{1}{2}U^{EX} \geq \frac{3}{4}U^r = \frac{3}{4}W^r$$

The first inequality follows from Lemma 5(1), and the second from Proposition 3(1). □

Proof of Lemma 2 The claim for $\alpha = 1$ follows by observing that $\mathrm{d}R^a/\mathrm{d}\alpha = \int_0^1 (x\psi'(x) - \psi(z))(1 - F(z))\mathrm{d}z > (<)0$ if ψ is convex (concave). The claim for $\alpha = 1/2$ follows by noting that $R_1^{EX} = EX(EY - EY_L) \geq \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] = R_{1/2}^{EX}$ follows by the same argument as the one used in the proof Proposition 6. □

Proof of Proposition 9 By Lemma 1, we know that $EY - EY_L \geq \frac{1}{2}EY$ if G is concave and ψ is convex. This yields:

$$\begin{aligned}
R_1^{EX} &= EX(EY - EY_L) \\
&\geq \frac{1}{2}EXEY \\
&= \frac{1}{2}U^r = \frac{1}{2}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\
&= \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}U^a \\
&\geq \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}R_1^a
\end{aligned}$$

The result follows then by noting that $CCV^2(X, \psi(X)) \leq 1$ if F and G are both IFR. □

Proof of Proposition 10 (1) We have the following chain:

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1 - F(EX)}(EX - EX_L)(EY - EY_L) \\
&\geq EXEY + (EX - EX_L)(EY - EY_L) \\
&= EXEY + R_1^{EX} - EX_L EY + EY_L EX_L \\
&= R_1^{EX} + EY(EX - EX_L) + EY_L EX_L \\
&\geq R_1^{EX} + \frac{1}{2}EXEY + EY_L EX_L \\
&= R_1^{EX} + \frac{1}{2}EXEY - \frac{1}{2}EXEY_L + \frac{1}{2}EXEY_L + EY_L EX_L \\
&= \frac{3}{2}R_1^{EX} + \frac{1}{2}EXEY_L + EY_L EX_L
\end{aligned}$$

The second line follows from Lemma 4(2) since F is concave, the third line is due to $R_1^{EX} = EX(EY - EY_L)$, the fifth line follows from Lemma 1(1). This implies that

$$W_1^{EX} = U^{EX} - R_1^{EX} \geq \frac{1}{3}U^{EX.} \geq \frac{1}{6}\left(U^a + U^r\right)$$
$$\geq \frac{1}{6}\left(U^a + \frac{1}{2}U^a\right) = \frac{1}{4}U^a \geq \frac{1}{2}W_{1/2}^a$$

where the second inequality follows from McAfee's result (recall the concave distributions are $DRFR$), the third inequality is due to the fact that $U^r \geq \frac{1}{2}U^a$ if F and G are both IFR, and the last inequality is due to the fact that $U^a < 2R_1^a$ if ψ is convex.

(2) If F, G are both convex, we use Lemma 1 to obtain:

$$R_1^{EX} = EX(EY - EY_L) \leq \frac{1}{2}U^r < \frac{1}{2}U^{EX}$$

This implies that

$$W_1^{EX} = U^{EX} - R_1^{EX} \geq \frac{1}{2}(U^a + U^r) - \frac{1}{2}U^r = \frac{1}{2}U^a \geq W_1^a$$

where the second inequality follows from McAfee's result since convex distributions are IFR. The last inequality follows from the same argument as the one at the end of part 1. □

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