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Innovation timing games: a general framework with applications☆

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Abstract

We offer a new algorithm for analyzing innovation timing games. Its main advantage over the traditional approach is that it applies to problems that had previously been intractable. We use the algorithm to examine two classical innovation problems. We find that the competition takes the form of a waiting game with a second-mover advantage either for any level of R&D costs (process innovation) or for high R&D costs (product innovation). Moreover, both models predict that the second-mover advantage is monotonically increasing in the costs of R&D.

JEL classification: L13; O31; O33

Keywords: Simple timing games; Preemption; Waiting; Innovation; R&D

1. Introduction

This paper studies the optimal timing of bringing a new product or process to the market. The timing decision is influenced by a basic trade-off: On the one hand, being first may yield monopoly profits till another firm enters the market. On the

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other hand, being late may lead to higher profits if late firms get access to better technology.[1] The main question is what determines the relative strengths of these two effects and how do the dynamics of such an interaction look like.

Our study of optimal innovation timing can be based on a fairly small literature in which technological competition is formulated as a simple timing game—i.e., a game in which each firm chooses at any point in time whether to make a single, irreversible move (cf. [5,6, Chapter 4.5; 4,12]).[2] Two important results of this literature are the following. First, in their extension of Reinganum's [16] duopoly model of technology adoption, Fudenberg and Tirole [5] show that a first-mover advantage is not supported by subgame-perfect strategies if firms are unable to precommit to future actions. In their model, firms decide at any point in time whether to adopt a cost-reducing new technology, knowing that adoption costs decline over time. By assumption, the increase in profits due to innovation is greater for the first adopter than for the second. As the authors show, this potential first-mover advantage stimulates preemption up to a point where the extra flow profit for the first mover just equals the extra costs of speeding up adoption. Second, Katz and Shapiro [12] demonstrate that a potential second-mover advantage may give rise to subgame-perfect equilibria in which preemption and payoff equalization do not occur. In their model, payoffs to different firms are asymmetric. Results for a symmetric setting are provided by Dutta et al. [4] who demonstrate that a potential second-mover advantage may indeed prevail as the subgame-perfect equilibrium outcome.

However, the approach to simple timing games analysis used in this literature requires rather restrictive assumptions. In particular, the first-mover's equilibrium payoff must be single-peaked in the times that firms may move first. If the possibility of multiple peaks of this payoff function cannot be excluded, the approach does not deliver a subgame-perfect equilibrium. We find that this approach is not applicable to many innovation timing problems with ongoing technological progress. The reason is that the first-mover's problem is typically complex since it incorporates the best response of the second mover, which is the solution to a non-trivial maximization problem.

The present paper aims to fill that gap by offering a more general approach for simple timing games which does not rely on ensuring single-peakedness of the first-mover's payoff function. The approach generalizes the existing results of Fudenberg and Tirole [5,6], Katz and Shapiro [12], and Dutta et al. [4]. Our central result is a theorem asserting general conditions under which simple timing games possess a unique equilibrium outcome. One property of this outcome is that, as long as the first-mover's equilibrium payoff is continuous in the times that firms may move first, it never involves a first-mover advantage. The competition is then structured either as a preemption game with payoff equalization in equilibrium or as a waiting game

[1] For empirical evidence, see, e.g., Tellis and Golder's [23] study of 50 different markets.

[2] This literature focuses on adoption of new technology. There is another, relatively large literature on technological competition, which focuses on invention of new technology, assuming that first discovery results in a patent which excludes others from innovating as well. Prominent examples include Loury [14], Lee and Wilde [13], Reinganum [15,17,19]. See also the discussion in [1]. For an analysis of technological competition when one firm is initially already in the market, see, e.g., [2].

with a second-mover advantage. We also provide an algorithm for determining whether a specific game is one of preemption or one of waiting. In addition, we offer two extensions for the analysis of simple timing games when the first-mover's payoff function is not necessarily continuous.

We illustrate the usefulness of our more general approach by applying it to two classical innovation scenarios: one of process innovation and another of product innovation. For process innovation, we consider a dynamic version of the classical process innovation story as in Dasgupta and Stiglitz [3] and Reinganum [16,18]. Firms choose a level of variable costs over time, which determines their cost position during subsequent Cournot quantity competition. For product innovation, we consider a dynamic version of a vertical product differentiation model adapted from Dutta et al. [4] and Tirole [24].[3] Firms choose at any date whether to bring the currently available product to the market or whether to wait and market a product of higher quality. Once both firms have entered, they compete in prices. To the best of our knowledge, the process innovation game has not been studied before. For the product innovation game, analytical results have been obtained so far only for polar cases [4,9]. Our analysis reveals that in both settings the competition may take the form of a waiting game with a second-mover advantage in equilibrium. Moreover, we find that in both games the second-mover advantage increases monotonically as R&D becomes more costly.

In Section 2, we describe the general framework as well as the specific process and product innovation timing games. In Section 3, we illustrate why the existing approach fails to provide solutions to these games. Also, in Section 3 the existence and uniqueness results for simple timing games are formally stated and proved. In Section 4, we apply our approach to the two innovation timing games to analyze the effects of R&D cost changes on the equilibrium innovation dynamics.

2. Simple timing games

2.1. Game form

We consider a class of simple timing games, Γ, characterized by the following structure: There are two firms, $i = a, b$. At any point in time $t \in \mathbf{R}_+$, each firm can choose whether to make an irreversible stopping decision, conditional on the history of the game. We will interpret the stopping decision as the firm's choice to adopt a currently available new technology. Let t_a, t_b denote the firms' respective adoption dates. A firm's payoff depends on its own and its rival's adoption date: $\pi_a(t_a, t_b)$ and $\pi_b(t_b, t_a)$. If firm i chooses t_1 while firm j $(j \neq i)$ chooses $t_2 > t_1$, then i is called the leader and j the follower. Throughout the paper, we will make the following basic assumptions:

(A1) Time is continuous in the sense of 'discrete but with a grid that is infinitely fine' (cf. Simon and Stinchcombe [22]). That is, any continuous-time strategy profile

[3] The model by Tirole [24] is based in turn on Shaked and Sutton [21].

will be restricted to an arbitrary, increasingly fine sequence of discrete-time grids, and the continuous-time outcome will be defined to be the limit of the discrete-time outcomes.[4]

(A2) There exist two piecewise continuous functions

$$\pi_1, \pi_2 : \{(t_1 \in \mathbf{R}_+) \times (t_2 \in \mathbf{R}_+) \mid 0 \leqslant t_1 \leqslant t_2\} \rightarrow \mathbf{R}_+$$

with $\pi_1(t_1, t_2) = \pi_2(t_1, t_2)$ if $t_1 = t_2$, and

$$\pi_i(t_i, t_j) = \begin{cases} \pi_1(t_i, t_j) & \text{if } t_i < t_j, \\ \pi_2(t_j, t_i) & \text{if } t_i \geqslant t_j \end{cases}$$

for $(i \neq j) \in \{a, b\}$.

(A3) If a firm is indifferent between the leader's and follower's role at any date t, then it attempts to become the leader. If, additionally, the leader is indifferent between adopting at two different points in time, then it chooses the earlier one. Furthermore, if $t_a = t_b$, then we assume that only one firm—each with probability 1/2—actually adopts at that time and becomes the leader, while the other firm becomes the follower and may postpone its adoption.[5]

Assumption A1 circumvents the problem that backwards induction cannot be applied in continuous time. That is, we regard discrete-time with a very fine grid as a convenient mathematical construction to represent the notion of 'continuous time'. Assumption A2 imposes symmetry between firms. Assumption A3 is used to formalize the idea that firms will be able to avoid coordination failure as an equilibrium outcome. That is, firms will not choose to move at the same instant of time if they would regret this move afterwards. As observed by Fudenberg and Tirole [5], an equilibrium involving a positive probability of coordination failure cannot be obtained in the polar case of a continuous-time game without a grid, where equilibria are defined to be the limits of discrete-time mixed-strategy equilibria. By contrast, in the limit of a discrete-time game where the period length converges to zero, coordination failure is a possible equilibrium outcome. Hence, if one uses a discrete-time game with very short time lags to represent the notion of 'continuous time' (as we do), one needs to make an assumption that *explicitly* rules out the possibility of coordination failure. Several alternative assumptions can be made: (i) a randomization device as it is used here, in [4,12,22]; (ii) alternating-moves as in [8,20]; (iii) firm-specific lags between observations and decisions as in [7].

[4] Simon and Stinchcombe [22] identify conditions under which the discrete-time outcomes converge to a unique limit that is independent of the particular sequence of grids. Roughly, the conditions require (i) an upper bound on the number of moves, (ii) that strategies depend piecewise continuously on time, and (iii) that actions later in the game are not "too sensitive" in a certain sense to the precise times at which earlier moves have been made. As it will turn out, the simple timing games considered in the present paper satisfy these conditions.

[5] In the limit, adoption by one firm may result in an *instantaneous* follow-on adoption by the other firm, i.e., the two firms adopt 'consecutively but at the same instant of time', and both firms obtain the same payoff.

2.2. Innovation timing games—two examples

In this subsection, we present two specific examples: first, process innovation timing and, second, product innovation timing. In both games, there are two firms who have the opportunity to develop a new product. At each point in time, each firm chooses whether to bring the new product to the market, using the so far developed technological potential for the rest of the game, or whether to continue to invest in research and development (R&D) to obtain a better technology. Let $k(t)$ be each firm's R&D costs per unit of time at time t.[6]

The monopoly profit per unit of time that is associated with the leader's entry at time t_1 is $R_M(t_1)$. The leader's and follower's equilibrium duopoly profits per unit of time as functions of the leader's and follower's entry times are $R_1(t_1, t_2)$, and $R_2(t_1, t_2)$, respectively. The respective payoffs are thus given by

$$\pi_1(t_1, t_2) = \int_{t_1}^{t_2} e^{-r\tau} R_M(t_1)\, d\tau + \int_{t_2}^{\infty} e^{-r\tau} R_1(t_1, t_2)\, d\tau - \int_0^{t_1} e^{-r\tau} k(\tau)\, d\tau, \tag{1}$$

$$\pi_2(t_1, t_2) = \int_{t_2}^{\infty} e^{-r\tau} R_2(t_1, t_2)\, d\tau - \int_0^{t_2} e^{-r\tau} k(\tau)\, d\tau \tag{2}$$

with $t_1 \leqslant t_2$, and $R_1(t_1, t_2) = R_2(t_1, t_2)$ if $t_1 = t_2$.

Process innovation timing: In the process innovation game, firms can choose to reduce the cost of producing the new product before they enter the market. The total cost of producing q_i units of output is $c_i q_i$ for firm $i = 1, 2$, where c_i is constant. These costs decline over time by means of a deterministic and possibly costly research technology: $c_i(t) = e^{-\alpha t_i}$, where $\alpha > 0$ is the rate of technological progress. Note that the cost-reducing technology is characterized by diminishing returns per period. A natural form for the R&D cost function in this context is $k = \lambda$, with $\lambda \geqslant 0$ for all t.[7]

The demand side for the new product is characterized by a simple linear inverse (flow) demand function. Money units are normalized such that inverse demand per unit of time is $p = 1 - q$ where q represents the aggregate quantity.

The equilibrium profit flows per unit of time for the monopolist and the duopolists from Cournot quantity competition with homogenous goods are, respectively:

$$\hat{R}_M = \tfrac{1}{4}(1 - c_1)^2,$$

$$\hat{R}_1 = \max\left\{\tfrac{1}{9}(1 - 2c_1 + c_2)^2, 0\right\},\ \hat{R}_2 = \min\left\{\tfrac{1}{9}(1 - 2c_1 + c_2)^2, \tfrac{1}{4}(1 - c_1)^2\right\}. \tag{3}$$

Note that there are two cases to distinguish: $c_2 \geqslant 2c_1 - 1$, i.e. the innovation by the follower is nondrastic, and $c_2 < 2c_1 - 1$, i.e. it is drastic. If the innovation becomes

[6] Notice that we allow for positive R&D costs per period of time, but ignore any additional fixed costs of market entry. As a consequence, at any date of market entry, all R&D expenditures are sunk. Furthermore, we assume that a firm that stops its R&D activity and is indifferent between entering the market at that date and staying out forever will choose to enter. Thus, entry deterrence is no issue in our paper.

[7] This form for the R&D cost function is, for instance, used by Reinganum [18].

drastic, the follower will set its monopoly price and the leader will shut down. Clearly, $\hat{R}_M(c_1)$ and $\hat{R}_i(c_1, c_2)$ can be written as functions of time. Hence the structure of the payoffs is of the form (1) and (2).

Product innovation timing: In the product innovation game, firms can choose to improve the quality of the new product before they enter the market. The available product quality $s(t)$ is increasing in time t by means of a deterministic and possibly costly research technology. We assume, as in [4], that s is proportional to t, and without further loss of generality that $t = s$. After a firm has entered the market, the quality of its product is fixed. Each firm's R&D costs per unit of time are λs, with $\lambda \geqslant 0$. Variable costs of production are independent of quality and zero.

For the demand side, we use a model adapted from Tirole [24]. Each period, each consumer buys at most one unit from either firm 1 or firm 2. Consumers differ in a taste parameter θ, and they get in each period a net utility if they buy a quality s_i at price p_i of $U = s_i\theta - p_i$, and zero otherwise. A consumer of "taste" θ will buy if $U \geqslant 0$ for at least one of the offered price-quality combinations, and she will buy from the firm that offers the best price-quality combination for her. Consumers are uniformly distributed over the range [0,1]. Without loss of generality, we choose physical and money units such that inverse (flow) demand $p(q)$ for s given quality units is: $p(q) = s(1 - q)$, where q denotes aggregate quantity.[8]

The equilibrium profit flows per unit of time for the monopolist and the duopolists from price competition with vertically differentiated goods are

$$R_M = \tfrac{1}{4} t_1,$$
$$R_1 = t_1 t_2 \frac{t_2 - t_1}{(4t_2 - t_1)^2}, \quad R_2 = 4t_2^2 \frac{t_2 - t_1}{(4t_2 - t_1)^2}, \tag{4}$$

respectively.

3. Solutions to simple timing games

The natural solution concept for simple timing games is subgame-perfect equilibrium. As in [22], we restrict attention to pure strategies and invoke the additional concept of iterated elimination of weakly dominated strategies. Here, the refinement is however only used to exclude some uninteresting, rather pathological cases.

Following Fudenberg and Tirole [5], we take the relevant choice variable to be the time that firms may choose to move first. Let $\mathfrak{R}(t) : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ be the best response function of the follower, defined on the set of times t that firms may choose to move first. If such a best response function $\mathfrak{R}(t)$ exists, we can specify the leader's and the follower's payoffs as functions of t alone: define $L(t) = \pi_1(t, \mathfrak{R}(t))$ and $F(t) = \pi_2(t, \mathfrak{R}(t))$, respectively.[9] To ensure that $\mathfrak{R}(t)$ is a well-defined function of the

[8] See [9] for a more detailed description of this example and for the analysis of the limiting case in which the R&D costs parameter λ tends to infinity.

[9] We use t instead of t_1 to denote the leader's adoption time whenever there is no ambiguity.

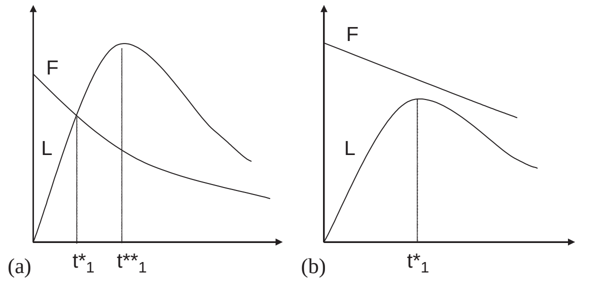

Fig. 1.

leader's choice t, we assume that if the follower is indifferent between moving on different dates, it will choose the earlier one.

The aim of this section is to isolate a class of simple timing games for which some equilibrium outcome can be uniquely identified and is easily described. We begin by briefly discussing the existing approach to simple timing games analysis.

3.1. Approach with single-peaked L curve

The existing literature on simple timing games (cf. [4,5], [6, Chapter 4.5; 12], requires that the leader payoff $L(t)$ satisfies one major assumption: L must have a unique maximum. The approach is suggested in Fig. 1.

Consider first the situation depicted in Fig. 1a. Suppose that the L curve is single-peaked at t_1^{**}. The solution can then be obtained by applying the following argument due to Fudenberg and Tirole [5].[10] It is clear that each firm would like to move first at t_1^{**}. Knowing this, however, each firm also has an incentive to preempt its rival by adopting slightly before t_1^{**}. Hence, first adoption at t_1^{**} cannot be an equilibrium. Similar reasoning can be applied to any $t_1 \in]t_1^*, t_1^{**}[$, where t_1^* denotes the intersection point between the L curve and the F curve. This yields first adoption at time t_1^* and equal payoffs for both firms as the unique subgame-perfect equilibrium outcome. Next, consider the situation depicted in Fig. 1b. Suppose that the L curve is single-peaked at t_1^*. Since the F curve lies above the L curve at any $t_1 \leqslant t_1^*$, it is clear that no firm has an incentive to preempt its rival before date t_1^*. In fact, the unique subgame-perfect equilibrium (up to relabelling of firms) involves first adoption at t_1^* and a higher payoff for the second mover.[11]

Note that one consequence of assuming that the L curve is single-peaked is that the problem of determining a ''terminal subgame of the game'', where one can begin

[10] In fact, a similar argument has already been made by Karlin [11, Chapter 6], however, without using the concept of subgame perfection.

[11] This equilibrium is asymmetric. That is, the competitors' expectations about the rival's strategies determine the equilibrium outcome. If, for example, firm i believes that j never enters first, i may choose to be the first entrant. Likewise, if j has the reputation of being likely to enter first, it may be optimal for i to wait until j has entered. In the case where the game is structured as a waiting game, there is also a continuum of mixed-strategy equilibria which are not considered here.

applying Fudenberg and Tirole's preemption argument, is simple: by examining the first-order condition for maximizing L, it is typically easy to determine the accurate location of the maximum of L. Then, what is left to check is whether the L curve is above or below the F curve at that point. If L is above F (as in Fig. 1a), the equilibrium involves preemption and payoff equalization. If L is below F (as in Fig. 1b), firms wait until the maximum point of the L curve, and there is a second-mover advantage in equilibrium.

In order to ensure that this approach is applicable, it is essential that the possibility of multiple peaks of the L curve can be excluded. If there were, say, two maxima of L, the equilibrium adoption date of the leader could be at the second maximum, but also at any earlier point in time. In such a case the approach used in the existing literature does not deliver a subgame-perfect equilibrium. Note that the failure to exclude multiple peaks of the L curve does not even allow for a numerical application of this approach. The reason is that for a numerical computation of the payoff curves it is essential to have an appropriate terminal condition. Otherwise, stopping the computation at some point after the equilibrium candidate t_1^* would not exclude the possibility of some $t' > t_1^*$ to be the leader's equilibrium adoption date instead of t_1^*.[12]

We argue that single-peakedness of the L curve cannot be regarded as a natural property of innovation timing games. In all applications that we have analyzed this assumption either had to be rejected or it turned out to be impossible to verify. The reason is that the first-mover's problem is typically complex since it incorporates the best response of the follower, which is the solution of a non-trivial maximization problem.

For the process innovation game, even for the simplest parameter constellations, that is for $\alpha = r = 1$, we detect multiple peaks of the L curve for $\lambda \in [0.0246, 0.0346]$.[13] An example is depicted in Fig. 2 where the thick curve is the L curve for $\alpha = r = 1$ and $\lambda = 0.03$. In this case, L has two peaks, one at $t = 0$ and another at $t = 0.717$.[14]

For the product innovation game described above, the best response of the follower, $t_2 = \Re(t_1)$, solves the following first-order condition:

$$r\lambda(4t_2 - t_1)^3 - 16t_2^2 + 12t_1t_2 - 8t_1^2 + 16rt_2^3 - 20rt_2^2t_1 + 4rt_2t_1^2 = 0 \tag{5}$$

which reveals that it is very difficult to exclude the possibility of multiple peaks of the L curve. In fact, we did not manage to analytically verify single-peakedness of L for this application. We show that the approach presented below can, however, easily be applied to determine the subgame-perfect equilibrium outcome of this game, since this approach does not rely on single-peakedness of L.

[12] Fudenberg and Tirole [5] allow for the possibility that the L curve may have a second peak, however, only in the range of joint adoption of leader and follower. They show that in that case there may exist several subgame-perfect equilibria.

[13] It can easily be checked that payoffs of either firm can only be positive for λ in the range $[0, 0.0625]$.

[14] The kinks of the L curve are due to a change in the follower's best response from drastic to non-drastic innovation. Additional complications arise in the case of $\alpha \neq r$, since not only the presence of multiple peaks and kinks of L are common phenomena, but the L curve exhibits also points of discontinuities for certain parameter constellations.

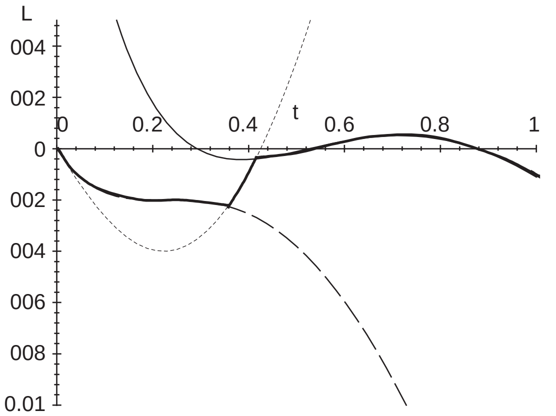

Fig. 2.

3.2. Approach with possibly multiple-peaked L curve

We now present an approach to simple timing games analysis that allows for the possibility that the L curve is not single-peaked. We shall also provide an algorithm for determining the dynamic nature of the game, i.e., whether it is a one of preemption or one of waiting.

Define date T_1 by

$$T_1 := \min\{\tau : L(\hat{t}_1(\tau)) \geqslant F(\tau)\}, \tag{6}$$

where

$$\hat{t}_1(\tau) := \max\left\{\hat{t} : \hat{t} = \arg\max_{[0,\tau]} L(x)\right\}, \tag{7}$$

That is, T_1 denotes the earliest point in time where the F curve just falls below the maximum value that the L curve achieves over the range $[0, t]$. In the next lemma, we state conditions under which a simple timing game $\gamma \in \Gamma$ has a unique point T_1. The lemma is proved in the appendix.

Lemma 1. *Assume that a simple timing game $\gamma \in \Gamma$ fulfills the following conditions*:

1. *There exists a best response function $\mathfrak{R}(t)$.*
2. *There exists some point $t' \in (0, \infty)$ such that $F(0) > L(0) \geqslant 0$ and $F(t') \leqslant 0$.*

Then there exists a unique T_1.

Our central result is a theorem asserting two rather mild conditions under which a simple timing game $\gamma \in \Gamma$ has a unique equilibrium outcome that is easily obtainable by analyzing the game only for the range $[0, T_1]$, irrespective of the shape of the payoff curves after T_1. We have incorporated these conditions in Fig. 3: (i) the L

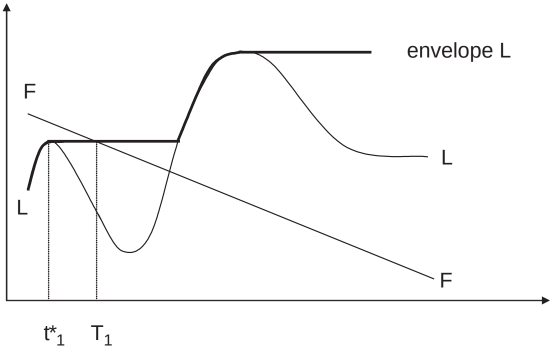

Fig. 3.

curve is continuous, and (ii) the F curve is continuous and non-increasing. In this figure, L has multiple peaks, which rules out an application of the existing approach. The thick curve, which we call the "envelope L curve", gives the maximum value that L achieves over the range $[0, t]$, for any given t. As we shall show below, under these two conditions of the payoff functions, point T_1 is the first intersection point between the envelope L curve and the F curve. Furthermore, this implies that T_1 is a boundary point of the set of times that firms will move first.[15]

Before we state our theorem, we will introduce the two conditions on the payoff functions informally and illustrate what happens if they are violated.

Continuous L curve: Fig. 4 illustrates the possible problem arising from a discontinuous L curve. In the figure, the L curve jumps upwards at some date later than T_1. One can verify that in this case there are at least two different subgame-perfect equilibrium outcomes: (i) both firms trying to be first at T_1, and (ii) both firms waiting until the point of discontinuity. Thus, discontinuities of the L curve may give rise to multiple equilibrium outcomes.

The condition is clearly restrictive, and especially not suitable if additional fixed-costs of market entry and hence entry deterrence are an issue.[16] However, apart from such cases, continuity of the L curve is typically satisfied in games with ongoing technological progress where the best response function of the follower, $\Re(t)$ does not change discontinuously, such as in the two innovation timing games considered in the present paper. Moreover, as we will show below, our algorithm may still be applicable when the condition is relaxed.

Continuous and non-increasing F curve: The second class of games that we exclude are those in which the F curve is discontinuous and/or increasing. Restricting F to be

[15] The approach of finding solutions via intersection point arguments is thus somewhat similar as the methodology used by Vives [25] in his analysis of supermodular games.

[16] Entry deterrence plays no role in the examples studied in this paper, since R&D expenditures are sunk at any date where the entry decision has to be made. Thus, entry is at any moment in time effectively costless. Also, note that no firm can force its rival into an eventually profitless situation, since any firm could decide to be the leader at any point in time. We leave the issue of entry deterrence for future research.

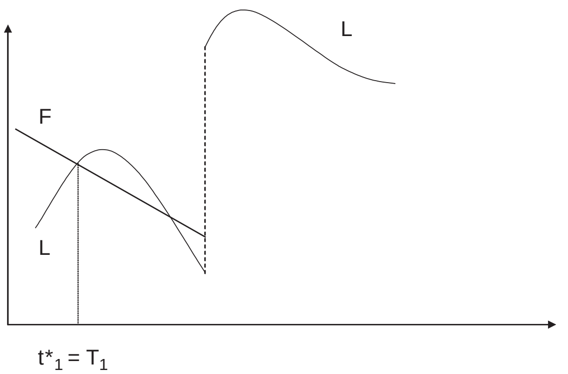

Fig. 4.

a continuous function, turns out to be a rather mild assumption, since this function gives the best-response payoff to any leader's choice t, and is obtained by integrating with respect to time some instantaneous flow profits, which typically depend only on time and the fact that the other firm has adopted already. Requiring F to be non-increasing in the leader's choice, is more restrictive. In the context of innovation timing, the assumption has, however, a rather natural interpretation: in the presence of ongoing technological progress, *earlier* innovation means usage of a *less advanced* technology. Thus, if the leader innovates earlier, it will be in a weaker technological position during product market competition. An earlier leader's choice thus implies *higher* duopoly profits for the follower. In fact, the assumption of a non-increasing F curve turns out to be typically satisfied for games of innovation timing where firms enter a new market, such as in the two examples considered in this paper.

Without the condition, one can still establish the existence of subgame-perfect equilibria. However, when the L curve has multiple peaks, the equilibrium outcome is not necessarily unique. The analysis of subgame-perfect equilibria then requires to examine a considerably larger set of subgames, with a "terminal node" specified in a way which depends on the particular nature of the specific game under consideration.

Theorem 1 (Unique equilibrium outcome). *Consider a simple timing game $\gamma \in \Gamma$ that satisfies the conditions stated in Lemma* 1 *and in addition*:

1. $L(t)$ *is continuous.*
2. $F(t)$ *is continuous and non-increasing.*

   Then the game has at least one subgame-perfect equilibrium in undominated pure strategies and the equilibrium outcome is unique (*up to relabelling of firms*): *one firm adopts at* t_1^*, *the other firm follows at* $\mathfrak{R}(t_1^*)$, *where* $t_1^* := \hat{t}_1(T_1)$, *as defined above*; *equilibrium payoffs are* $L(t_1^*) = F(t_1^*)$ *if* $t_1^* = T_1$, *and* $L(t_1^*) < F(t_1^*)$ *if* $t_1^* < T_1$.

Proof. In this proof, we will use the following joint definition:

$$\tilde{T}_1(t) \equiv \min\{\tau:\ L(\tilde{t}_1(t,\tau)) \geqslant F(\tau)\}, \tag{8}$$

$$\tilde{t}_1(t,\tau) := \max\left\{\hat{t} : \hat{t} = \arg\max_{[t,\tau]} L(x)\right\}. \tag{9}$$

That is, $\tilde{T}_1(t)$ denotes the point in time where the F-curve just falls below the maximum level that the L-curve obtains from some t up to $\tilde{T}_1$. Correspondingly $\tilde{t}_1(t,\tilde{T}_1)$ is the point in time where the L-curve attains that maximum.

Note that $T_1 = \tilde{T}_1(0)$ and $t_1^* = \tilde{t}_1(0, T_1)$, where T_1 is as defined above. By Lemma 1, t_1^* and T_1 exist and are unique.

We show that a game $\gamma \in \Gamma$ that satisfies the conditions of Lemma 1 has a subgame-perfect equilibrium consisting of the following pair of pure strategies:

Given no previous adoption, both firms choose "No adoption" at any $t < t_1^*$. At any $t \geqslant t_1^*$, given no previous adoption, firm i chooses

- "Adoption" if $L(t) \geqslant F(t)$ or $[L(t) < F(t)$ and $t = \tilde{t}_1(t, \tilde{T}_1(t))]$, or
- "No adoption" if $[L(t) < F(t)$ and $t \neq \tilde{t}_1(t, \tilde{T}_1(t))]$,

and firm j chooses

- "Adoption" if $L(t) \geqslant F(t)$, or
- "No adoption" if $L(t) < F(t)$.

If either firm has already adopted at some time τ, the other firm adopts at $\Re(\tau)$.

First, note that the conditions stated in Lemma 1 ensure that each firm finds it optimal to wait at any $t < t_1^*$, but that adoption will eventually occur at some $t \geqslant t_1^*$ with $t < \infty$. Next, consider the subgames starting at any $t \geqslant t_1^*$. There are three cases to check for profitable deviations:

(i) If $L(t) \geqslant F(t)$, the given strategies yield a payoff of $1/2[L(t) + F(t)]$ for each firm, while any deviation yields at most $F(t)$, with $F(t) \leqslant 1/2[L(t) + F(t)]$.

(ii) If $L(t) < F(t)$ and $L(\tilde{t}_1(t, \tilde{T}_1(t))) < F(\tilde{t}_1(t, \tilde{T}_1(t)))$, the given strategies yield a payoff of $L(\tilde{t}_1(t, \tilde{T}_1(t)))$ for firm i and a payoff of $F(\tilde{t}_1(t, \tilde{T}_1(t)))$ for firm j. We now show that any deviation of firm i yields at most $L(\tilde{t}_1(t, \tilde{T}_1(t)))$, given j's strategy. The only possibly profitable deviation must involve "*Adoption*" at some $t' > \tilde{T}_1(t)$ with $L(t') > L(\tilde{t}_1(t, \tilde{T}_1(t)))$. It follows from Conditions 1 and 2 that in that case there must be an intersection between L and F at some t'' for $\tilde{T}_1(t) < t'' < t'$. However, the strategy of firm j prescribes "*Adoption*" at t''. This yields $F(t'') = L(t'')$ for firm i, which is smaller than $L(\tilde{t}_1(t, \tilde{T}_1(t)))$ by Condition 2; a contradiction. Note further that any deviation of firm j yields at most $1/2[L(\tilde{t}_1(t, \tilde{T}_1(t))) + F(\tilde{t}_1(t, \tilde{T}_1(t)))]$, given i's strategy, with $1/2[L(\tilde{t}_1(t, \tilde{T}_1(t))) + F(\tilde{t}_1(t, \tilde{T}_1(t)))] < F(\tilde{t}_1(t, \tilde{T}_1(t)))$.

(iii) If $L(t) < F(t)$ and $L(\tilde{t}_1(t, \tilde{T}_1(t))) = F(\tilde{t}_1(t, \tilde{T}_1(t)))$, the given strategies yield a payoff of $L(\tilde{t}_1(t, \tilde{T}_1(t))) = F(\tilde{t}_1(t, \tilde{T}_1(t))$ for each firm. Any "*Adoption*" before

$\tilde{t}_1(t, \tilde{T}_1(t))$ is weakly dominated by "*No adoption*" until $\tilde{t}_1(t, \tilde{T}_1(t))$. On the other hand, no firm can gain from waiting longer than $\tilde{t}_1(t, \tilde{T}_1(t))$.

Thus, the described strategies are best responses to each other and constitute a subgame-perfect equilibrium in every simple timing game $\gamma \in \Gamma$ that satisfies Conditions 1 to 2. If the strategies are played on an arbitrary discrete-time grid, the resulting equilibrium outcome is that the first adoption occurs weakly beyond t_1^*. In the limit, firms adopt exactly at $(t_1^*, \Re(t_1^*))$, with $L(t_1^*) = F(t_1^*)$ if $t_1^* = T_1$ and $L(t_1^*) < F(t_1^*)$ if $t_1^* < T_1$.

To prove the second claim, we show that there exists no subgame-perfect equilibrium in pure strategies which does not implement first adoption at t_1^* and equilibrium payoffs of $L(t_1^*) = F(t_1^*)$ if $t_1^* = T_1$, and $L(t_1^*) < F(t_1^*)$ if $t_1^* < T_1$. Note first that the strategies given above are a subgame-perfect equilibrium for $i = a$ and $j = b$, and vice versa, and, moreover, for relabeling of the firms in subgames starting at any $\bar{t}$ where $L(\bar{t}) = F(\bar{t})$. Clearly, such relabeling in subgames starting at any $t > t_1^*$ does not change the equilibrium outcome since t_1^* remains the leader's choice in the described equilibrium.

Consider now potential equilibria with first adoption at t_1 where $t_1 > t_1^*$. By Condition 2, we must have for any $t_1 > t_1^*$ that either $F(t_1) \leqslant L(t_1)$ or $L(t_1) < L(t_1^*)$. Clearly, if $L(t_1) < L(t_1^*)$, $t_1 > t_1^*$ cannot be the leader's choice in an equilibrium in pure strategies. Now consider the case where $F(t_1) < L(t_1)$ and $L(t_1) \geqslant L(t_1^*)$. In that case t_1 cannot be the leader's choice in equilibrium either, since, if one firm attempts to become the leader at that date, it is always profitable for the other firm to become the leader slightly earlier. Note that the case where $F(t_1) = L(t_1)$ and $L(t_1) \geqslant L(t_1^*)$ for any $t_1 > t_1^*$ is ruled out by assumption (A3).

Finally, consider possible equilibria with first adoption at t_1 where $t_1 < t_1^*$. Note that by definition of t_1^*, the inequalities $L(t) \leqslant L(t_1^*)$ and $F(t_1) > L(t_1)$ must hold for all $t_1 < t_1^*$. Hence, "*Adoption*" by one firm before t_1^* is a weakly dominated strategy. This completes the proof of the theorem. □

The equilibrium strategies identified in the proof of Theorem 1 have the following properties. The first adoption occurs at point $t_1^* \leqslant T_1$, and the second adoption at point $\Re(t_1^*) \geqslant t_1^*$. If $t_1^* = T_1$, firms engage in a preemption game to be the first, and they obtain equal payoffs, i.e., $L(t_1^*) = F(t_1^*)$. If $t_1^* < T_1$, firms engage in a waiting game to be the best, and there is a second-mover advantage, i.e., $L(t_1^*) < F(t_1^*)$.

Note that our results reduce to those of the existing literature if the theorem is applied to problems where the L curve is ensured to be single-peaked. For example, in the situations depicted in Fig. 1, we have $T_1 = t_1^*$, so we know from Theorem 1 that the unique equilibrium outcome involves preemption and first innovation at t_1^*, with payoff equalization across firms. In Fig. 1b, we have $T_1 > t_1^*$, so the unique equilibrium outcome involves waiting until one firm innovates at t_1^*, and there is a higher payoff for the second mover. Theorem 1 thus generalizes the existing results by omitting the usual assumption on the first-mover's equilibrium payoff. Furthermore, the theorem isolates a class of simple timing games for which the equilibrium outcome is unique and easily describable, thus providing a basis for numerical approaches to compute explicit solutions to particular games.

3.3. Approach with possibly discontinuous L curve

This subsection shows that the application of our algorithm is not restricted to games with a continuous L curve. The following two corollaries to Theorem 1 are proved in the appendix.

Corollary 1. *Consider a simple timing game $\gamma \in \Gamma$ that satisfies the conditions stated in Lemma* 1 *and in addition*:

1. *Let $Q = \{t : L(t) \geqslant F(t)\}$. Then $L(\sup X) \leqslant \sup L(X)$ and $L(\inf X) \geqslant \inf L(X)$ for every nonempty subset X of Q.*
2. *$F(t)$ is continuous and non-increasing.*

Then the unique equilibrium outcome is as described in Theorem 1.

Corollary 1 reveals that the algorithm suggested in Theorem 1 continues to be applicable to simple timing games in which the L curve is discontinuous, but involves no upwards jumps above the F curve. Note that in this case the envelope L curve is still continuous for $t \geqslant T_1$. This implies that point T_1, as defined above, is still a boundary point of the set of times that firms will move first.

In the next corollary, we deal with cases where the envelope L curve is not necessarily continuous for $t \geqslant T_1$.

Corollary 2. *Consider a simple timing game $\gamma \in \Gamma$ that satisfies the conditions stated in Lemma* 1 *and Condition* 2 *of Theorem* 1. *In addition assume* A3(ii) *that firms move alternately, first a then b, then a again and so on* (*as for example in* [8,20]). *Then the equilibrium outcome is either*:

1. *as described in Theorem* 1: *one firm adopts at t_1^*, the other firm follows at $\Re(t_1^*)$, where $t_1^* := \hat{t}_1(T_1)$, as above*; *equilibrium payoffs are $L(t_1^*) = F(t_1^*)$ if $t_1^* = T_1$, and $L(t_1^*) < F(t_1^*)$ if $t_1^* < T_1$; or*
2. *one firm adopts at t_1^*, the other firm follows at $\Re(t_1^*)$, where $t_1^* = T_1$; equilibrium payoffs are $L(t_1^*) > F(t_1^*)$.*

Corollary 2 shows that possible equilibrium candidates involving first adoption beyond point T_1 are not robust to a change in the alternate A3 assumptions used for ruling out coordination failure as a possible equilibrium outcome. Thus, in games where the envelope L curve is not a continuous function, the equilibria involving first adoption at or before T_1 are the only equilibria that have an alternate-move/discrete-time analog. These equilibria are captured by the algorithm suggested in Theorem 1.

4. Applications

In this section, we illustrate the usefulness of our more general approach by applying it to the two innovation timing games described above. In particular, we wish to identify the dynamic nature of these games and study the effects of changing the R&D costs on the firms' timing incentives and equilibrium payoffs. The numerical treatment is described in Appendix B.

4.1. Process innovation timing

For the process innovation game described in Section 2.2, it is easy to verify that the follower payoff π_2 is not single-peaked with respect to t_2. This implies the possibility of discontinuous changes of the follower best response and hence possible discontinuities of the L curve. However, as we show in the following proposition, when the rate of technological progress α is equal to the rate of time preference r, no such discontinuities occur. Hence, Theorem 1 is applicable.[17]

Proposition 1. *The described process innovation game satisfies the conditions of Theorem* 1 *if* $\alpha = r$.

Proof. First we check whether the conditions stated in Lemma 1 are satisfied. Note that the first condition in Lemma 1 is satisfied by assumption. Furthermore, one can easily check that $L(0) < F(0)$ holds. Thus, to verify that the second condition in Lemma 1 is satisfied, it is sufficient to show that $\lim_{t_1 \to \infty} F \leqslant 0$. For this let $\pi_M = \int_t^\infty R_M e^{-r\tau}\, d\tau - \int_0^t e^{-r\tau} k(\tau)\, d\tau$ be the payoff of a monopolist innovating at time t. Clearly, for $\hat{t}$ large enough, we have $F(t) < \pi_M(t)$ for all $t > \hat{t}$. Hence, it suffices to show that $\lim_{t \to \infty} \pi_M \leqslant 0$. This in turn follows from $\lim_{t \to \infty} \int_t^\infty (\frac{1}{4}(1 - e^{-t})^2) e^{-r\tau}\, d\tau = 0$.

By Lemma A.1, which is stated and proved in the appendix, we know that $\mathfrak{R}$ is continuous. Since π_1 is continuous in both arguments, Condition 1 of Theorem 1 is satisfied.

We now check whether Condition 2 of Theorem 1 is satisfied. It is clear that F is continuous, with $F' = \partial\pi_2/\partial t_1$, since $\mathfrak{R}$ is best response, and $\partial\pi_2/\partial t_1 = 1/r \cdot e^{-rt_2} \cdot \partial R_2/\partial t_1$. Since $\partial R_2/\partial t_1$ is either negative or zero here, we obtain that F is non-increasing. $\square$

Thus, by Theorem 1, we know that the game has a unique equilibrium outcome for $\alpha = r$. Furthermore, it is sufficient to evaluate the L and F functions for the range of $[0, T_1]$ in order to be able to characterize this outcome. The results of our analysis

[17] The more general case of $\alpha \neq r$ can be investigated numerically by applying Corollary 2. The numerical analysis is however rather complex and beyond the scope of this paper. Preliminary results for this case imply that our findings for $\alpha = r$, as presented here, are fairly robust.

are presented in Table 1, where $L^* := L(t_1^*)$ and $F^* := F(t_1^*)$.[18] For all values of the R&D cost parameter λ, we obtain that $t_1^* < T_1$. That is, the competition is always structured as a waiting game with a second-mover advantage in equilibrium. This indicates that both firms value the strategic advantage of being the low-cost firm during product market competition more than the temporary monopoly position obtainable for the first innovator. Preemption and payoff equalization do not occur in this game.

Furthermore, we find that the second-mover advantage, as measured by the ratio of the follower equilibrium payoff to the leader equilibrium payoff, is monotonically increasing in the cost of R&D, λ. This monotonicity in the cost of R&D may come as a surprise. After all, the follower firm must pay R&D expenditures for a longer period of time than the leader. However, apart from this direct effect, an increase in the R&D costs per unit of time has the following two indirect effects. First, the follower's innovation occurs earlier, reducing the duration of the leader's monopoly period. Second, the leader's innovation occurs earlier as well. This means that the leader adopts a less advanced technology, which has a positive impact on the duopoly profits of the follower $(\partial R_2/\partial t_1 < 0)$. Thus, we may conclude that these indirect effects outweigh the direct effect.

Finally, it is interesting to note that for very high λ the leader effectively stays away from the market, while the follower becomes a monopolist.[19]

The approach offered in this paper may also be used to evaluate the impact of specific R&D policies. Measuring welfare as the present value of the sum of firms' equilibrium payoffs, L^* and F^*, the intertemporal stream of consumer surplus in monopoly and duopoly, respectively, and tax revenue, we find for the process innovation game described in Section 2.2 that the appropriate policy depends critically on the magnitude of the second-mover advantage in equilibrium: Taxation of R&D is found to be welfare-enhancing if the second-mover advantage is small, and subsidization if it is large. Moreover, if R&D costs are so high that no firm would innovate at all, we find that a subsidy, inducing one of them to innovate, and consequently monopolize the industry, can improve social welfare.

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[18] Without loss of generality, we have chosen the units of time such that $r = 1$.

[19] For the limiting cases, $\lambda = 0$ and $\lambda \to 0.0625$ (and $\alpha = r$), it is possible to confirm the numerical results analytically. Consider first the case where λ becomes large. It is straightforward to check that for $\lambda > 0.0625$ not even a monopolist could earn positive profits by innovating at the optimal point in time. Clearly, as λ becomes large (but still smaller than 0.0625) the only possible equilibrium outcome is that one firm adopts at the optimal point in time, while the other firm stays out of the market (or "adopts" at $t = 0$). Applying Theorem 1, we know that in fact this is an equilibrium outcome. That is, one firm "adopts" at $t = 0$ (i.e., the leader), and hence stays out of the market effectively, while the other firm (i.e., the follower) adopts later at the optimal point in time and earns monopoly profits forever after (i.e., a waiting game structure with a second mover advantage). Now consider the case of $\lambda = 0$. It is possible to show that both, the L curve and the F curve, written as functions of the leader's choice of variable costs, c_1, are a composition of three polynomials of the third degree. It is somewhat tedious but straightforward to verify analytically that we have a waiting game structure with a second mover advantage in that case as well, as is stated in Table 1.

Table 1
Results for process innovation

λ	t_1^*	c_1^*	t_2^*	c_2^*	T_1	L^*	F^*	**L*/F***
0.000	1.017	0.362	1.483	0.227	1.058	0.020	0.021	**96.81%**
0.001	1.012	0.363	1.478	0.228	1.056	0.020	0.020	**96.54%**
0.002	1.004	0.366	1.469	0.230	1.053	0.019	0.020	**95.98%**
0.003	0.994	0.370	1.458	0.233	1.048	0.018	0.019	**95.33%**
0.004	0.985	0.373	1.448	0.235	1.044	0.018	0.019	**94.64%**
0.005	0.976	0.377	1.439	0.237	1.040	0.017	0.018	**93.93%**
0.010	0.932	0.394	1.392	0.249	1.019	0.013	0.015	**89.34%**
0.020	0.831	0.436	1.297	0.273	0.967	0.006	0.009	**68.96%**
0.030	0.717	0.488	1.210	0.298	0.911	0.001	0.005	**10.93%**
0.031	0.000	1.000	0.908	0.403	0.889	0.000	0.017	**0.00%**
0.062	0.000	1.000	0.697	0.498	0.316	0.000	0.000	**0.00%**
0.064	0.000	1.000	0.000	1.000	0.000	0.000	0.000	—

4.2. *Product innovation timing*

The following proposition shows that Theorem 1 is applicable to the model of product innovation timing described in Section 2.2.

Proposition 2. *The described game of product innovation satisfies the conditions of Theorem* 1.

Proof. By the same arguments as in the proof of the previous proposition it is ensured that the conditions in Lemma 1 are satisfied.

Now we check Condition 1 of Theorem 1. Note that continuity of the follower best response function $\mathfrak{R}$ is a sufficient condition for Condition 1 of Theorem 1. To show that $\mathfrak{R}$ is continuous, we will show that π_2 is single-peaked in t_2 for any t_1 which in turn is satisfied if it is ensured that π_2 cannot have a local minimum in t_2 for $t_2 > t_1$. Note that π_2 is twice differentiable. In fact we have

$$\frac{\partial^2 \pi_2}{\partial t_2^2} = -r \cdot \frac{\partial \pi_2}{\partial t_2} + e^{-t_2 r} \cdot \left(\frac{1}{r} \cdot \frac{\partial^2 R_2}{\partial t_2^2} - \frac{\partial R_2}{\partial t_2} - k' \right)$$

with $\partial^2 R_2/\partial t_2^2 < 0$ and $\partial R_2/\partial t_2 > 0$. Hence $\partial^2 \pi_2/\partial t_2^2$ is strictly negative at point where $\partial \pi_2/\partial t_2 = 0$. This ensures that π_2 cannot have a local minimum and consequently is single-peaked in t_2.

Next, we check Condition 2 of Theorem 1. Clearly F is continuous. To show that F is non-increasing, note that $F' = \partial \pi_2/\partial t_1$, since $\mathfrak{R}$ is best response. We have $\partial \pi_2/\partial t_1 = 1/r \cdot e^{-rt_2} \cdot \partial R_2/\partial t_1$ which is negative since $\partial R_2/\partial t_1$ is negative everywhere. □

By Proposition 2, the game has a unique equilibrium outcome. The numerical results are presented in Table 2.[20] If R&D costs, λ, are low, we find that $t_1^* = T_1$, i.e.,

[20] Clearly, the results are unaffected by a normalization of the units of time. Hence, as in the process innovation game, we have normalized, without loss of generality, units of time such that $r = 1$.

Table 2
Results for product innovation

λ	t_1^*	t_2^*	T_1	L^*	F^*	**L*/F***
0.00	0.710	1.573	0.710	0.057	0.057	**100.0%**
0.01	0.688	1.501	0.688	0.053	0.053	**100.0%**
0.02	0.670	1.443	0.671	0.050	0.050	**100.0%**
0.03	0.651	1.380	0.651	0.047	0.047	**100.0%**
0.04	0.633	1.325	0.633	0.044	0.044	**100.0%**
0.05	0.619	1.281	0.619	0.041	0.041	**100.0%**
0.07	0.571	1.176	0.589	0.036	0.037	**96.6%**
0.10	0.479	1.024	0.547	0.030	0.036	**84.6%**
0.20	0.297	0.714	0.427	0.018	0.032	**57.2%**
0.50	0.125	0.377	0.242	0.007	0.024	**28.7%**
10.21	0.005	0.024	0.016	0.000	0.002	**7.3%**
∞	0.000	0.000	0.000	0.000	0.000	**6.3%**

the competition takes the form of a preemption game, with equal payoffs for both firms in equilibrium. However, if λ gets high, we obtain $t_1^* < T_1$, and there is a second-mover advantage in equilibrium. That is, the dynamic nature switches from a preemption game to a waiting game as R&D becomes more costly. Moreover, the second-mover advantage is monotonically increasing in the costs of R&D,[21] just like in the process innovation game. This suggests that the direct effect of higher R&D costs is outweighed by the indirect effects on the duration of the leader's monopoly period and the follower's duopoly profits, similarly as described above for the process innovation game. Finally, our welfare analysis of the product innovation game suggests that an R&D subsidy always leads to higher welfare.

Appendix A

Proof of Lemma 1. The first condition ensures that functions $L(t)$ and $F(t)$ exist. The boundary condition ensures that the set of points $\{\tau : L(\hat{t}_1(\tau)) \geqslant F(\tau)\}$ is non-empty. This implies that T_1, as defined by (6), exists. Note that T_1 is unique by definition. □

Lemma A.1. *If $\alpha = r$ and λ is small enough such that a monopolist can earn positive profits, then the follower's best response $\mathfrak{R}$ is continuous.*

Proof of Lemma A.1. We give only a sketch of the proof which is straightforward but long and tedious. The details are available from the authors on request.

[21] In [9], we have shown analytically for the limiting cases of $\lambda = 0$ and $\lambda \to \infty$ that the game is of a preemption type in the former case and of a waiting game type in the latter case. It follows from continuity arguments that there exists a threshold level of λ such that the game moves from the preemption game scenario to the waiting game scenario, as indicated by the numerical results in Table 2.

Note that we can write the follower's payoff function as a function of c_2

$$\pi_2(c_2) = \min(\pi^{\text{nd}}, \pi^{\text{d}}) \tag{10}$$

defined over the domain $[0, c_1]$ where

$$\pi^{\text{nd}}(c_2) = 1/9 \cdot (1 - 2c_2 + c_1)^2 c_2 - (1 - c_2)\lambda \tag{11}$$

is the follower's payoff in case of a non-drastic innovation, i.e. if $c_2 \geqslant 2c_1 - 1$;

$$\pi^{\text{d}}(c_2) = 1/4 \cdot (1 - c_2)^2 c_2 - (1 - c_2)\lambda \tag{12}$$

is the follower's payoff in case of a drastic innovation, i.e. if $c_2 \leqslant 2c_1 - 1$. Note that $c_2 = e^{-rt_2}$, since $\alpha = r$. Furthermore, we have normalized (without further loss of generality) r to 1. We use the following observations:

1. The minimum and the maximum of π^{nd} are at $c_{\text{min-nd}} = \frac{1}{3} + \frac{1}{3}c_1 + \frac{1}{6}\sqrt{1 + 2c_1 + c_1^2 - 27\lambda}$ and $c_{\text{max-nd}} = \frac{1}{3} + \frac{1}{3}c_1 - \frac{1}{6}\sqrt{1 + 2c_1 + c_1^2 - 27\lambda}$, respectively;
2. the minimum and the maximum of π^{d} are at $c_{\text{min-d}} = \frac{2}{3} + \frac{1}{3}\sqrt{1 - 12\lambda}$ and $c_{\text{max-d}} = \frac{2}{3} - \frac{1}{3}\sqrt{1 - 12\lambda}$, respectively;
3. the only possible intersection of π^{nd} and π^{d} is at $c_{\text{int}} = 2c_1 - 1$, where for $c_2 \leqslant c_{\text{int}}$ it is ensured that $\pi^{\text{d}} \leqslant \pi^{\text{nd}}$ while for $c_{\text{int}} \leqslant c_2$ it is ensured that $\pi^{\text{nd}} \leqslant \pi^{\text{d}}$;
4. the payoff function of a monopolist is equal to $\pi^{\text{d}}(c_2)$, and a monopolist can earn positive payoffs if and only if $\pi^{\text{d}}(c_{\text{max-d}})$ is positive which in turn is true if and only if $\lambda < \lambda_{\max} := 1/16$.

The proof proceeds in two main steps: First we show that π_2 is single-peaked over the interval $[0, c_1]$ as long as $c_1 \leqslant c_1^{\text{crit}} := \frac{3}{5} + \frac{1}{20}\sqrt{19} \simeq 0.82$, and $\lambda \leqslant \lambda_{\max}$. We do so by showing (i) that $c_{\text{min-nd}} > c_1$ and further (ii) that $c_{\text{min-d}} > c_{\text{int}}$. The second main step is to show that the global maximum of π_2 over the interval $[0, c_1]$ is at $c_{\text{max-d}}$ for all $c_1 \geqslant c_1^{\text{crit}}$. This is shown in four sub-steps:

1. We show that $0 < c_{\text{max-nd}} < c_{\text{int}} < 1$.
2. We show that $0 < c_{\text{max-d}} < c_{\text{int}} < 1$.
3. We show that $\pi^{\text{d}}(c_{\text{max-d}}) > \pi^{\text{d}}(c_{\text{int}})$.
4. We show that $\pi^{\text{d}}(c_{\text{max-d}}) > \pi^{\text{nd}}(c_1)$.

Proof of Corollary 1. It is not difficult to verify that the arguments of the proof of Theorem 1 continue to be applicable to a simple timing game $\gamma \in \Gamma$ that satisfies the conditions stated in Lemma 1 and Condition 2 of Theorem 1, provided the envelope L curve is continuous for $t \geqslant T_1$. Clearly, condition 1 of the corollary is sufficient for the envelope L curve to be continuous for $t \geqslant T_1$. □

Proof of Corollary 2. It is obvious that the equilibrium identified in Theorem 1 continues to be an equilibrium under the alternate-move assumption A3(ii) instead of A3(i) in Section 2.1, except that the leader and follower role are uniquely determined in equilibrium in the case of $t_1^* = T_1$.

Apart from A3(ii), the only difference between the corollary and Theorem 1 is that here we allow for the case that the envelope L curve is discontinuous. Note that the preemption argument as formulated in the proof of Theorem 1 continues to hold under the hypotheses of the corollary as well, except at some possible points of discontinuity of the envelope L curve. Hence, the only possible equilibrium candidates apart from those formulated in Theorem 1 involve both firms waiting until some point of discontinuity, say $\ddot{t}$. Note that $\ddot{t} \geqslant T_1$ by the definition of T_1.

We now show that waiting until $\ddot{t}$ cannot be an equilibrium if $\ddot{t} > T_1$. Suppose both firms wait until that date. Then the firm which is first to move at or after $\ddot{t}$, will do so, while the other firm is sure to become the follower with strictly lower profits than are obtainable by moving earlier.

Next we consider the case where $\ddot{t} = T_1$. Clearly, by the definition of T_1, the only possible equilibrium involves waiting until $\ddot{t}$. By assumption A3(ii), the firm which is first to move at or after $\ddot{t}$, will do so, while the other firm is sure to become the follower with strictly lower profits than the first firm. □

Appendix B

Description of numerical treatment

For our approach to simple timing games analysis, it is necessary to evaluate the L curve and the F curve with high accuracy over the whole relevant domain $[0, T_1]$. In the applications considered in this paper, we used a step size for t_1 of $T_1 \times 10^{-6}$. Since we had to evaluate the best response of the follower for each value of $t_1 \in [0, T_1]$, we needed to evaluate, for each parameter constellation, 10^6 times the arg max of the follower's profit $R(t_1) := \arg\max_{t_2} \pi_2(t_1, t_2)$.

In both applications, it was possible to compute the formula for the best response directly. In the case of product innovation, the problem of finding the follower's best response is equivalent to the problem of solving a polynomial of third degree with respect to t_2. Alternatively, since $\pi_2(t_1, t_2)$ can shown to be single-peaked in t_2, we could have used a standard bi-section algorithm, as described in Judd [10], to find the only possible root of $\partial\pi_2/\partial t_2$. The bi-section algorithm would need between 25 and 35 loops to reach an accuracy goal of $\varepsilon = (\hat{t}_2 - R(t_1)) \times 10^{-7}$ with $R(t_1) \in [\hat{t}_2 - \varepsilon, \hat{t}_2 + \varepsilon]$, where $\hat{t}_2$ is the reported solution and $\Re(t_1)$ the true best response. Note that this bi-section algorithm is applicable whenever it is ensured that π_2 is single-peaked in t_2, even if it is not possible to find the possible roots of $\partial\pi_2/\partial t_2$ analytically. The calculation time by using the bi-section algorithm instead of calculating the solution of the third-degree polynomial directly is about twice as high.

In the case of process innovation, finding the best response is equivalent to solving a quadratic equation with respect to c_2. This is true for any α, λ, i.e., not only for $\alpha = r$ as reported in Section 4.

We used a 32-Bit Pascal implementation (FreePascal) of this numerical procedure, using extended format for the floating-point numbers. The calculation time for one parameter constellation was always less than 1 minute on a standard (500 MHz) PC. The program code as well as the numerical results for $\alpha \neq r$ (and hence possible discontinuities) are available from the authors on request.

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