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Multiple prizes in research tournaments

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HIGHLIGHTS

- Research tournaments can award single big prizes or multiple smaller ones.
- With multiple prizes, research tournaments and sequential contests are equivalent.
- In research tournaments, one big prize is more effective than several small ones.

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ABSTRACT

A Fullerton–McAfee research tournament with multiple prizes is strategically equivalent to a sequential multi-prize Tullock contest. Contest designers, aiming to maximize total research efforts, should therefore allocate a given prize sum to a single prize rather than to several ones.

1. Introduction

In the past two decades, a number of so-called mega prizes with cash values of $1 million or more have been launched for research, among them the Queen Elizabeth Prize in Engineering, the Global Energy International Prize, the Gates Award for Global Health, or the Gotham Prize for Cancer Research. While the scientific community has generally welcomed the attending recognition of the importance of science, some researchers have questioned whether such large single prizes are the best way to promote research activities (see, e.g. Merali, 2013; Cha, 2016). In particular, should a contest designer, wishing to maximize total research efforts or the quality of the best discovery, allocate the entire prize sum to only one "first" prize?

Previous work on prize allocation in contests has delineated circumstances for the optimality of either a single prize or multiple prizes (e.g., Clark and Riis, 1998; Moldovanu and Sela, 2001; Sisak, 2009; Akerlof and Holden, 2012; Schweinzer and Segev, 2012). The issue has, however, not been analyzed in the classical research tournament model of Fullerton and McAfee (1999) — a surprising fact, given the important applications of this model to academic practice (Terwiesch and Xu, 2008; Liotard and Revest, 2018).

In the Fullerton–McAfee model, a number of researchers exert efforts to generate an output of random quality. Efforts are modeled as independent draws from a continuous distribution where greater efforts yield a higher likelihood of obtaining a high-quality output.[1] In the original setting, a single prize is awarded to the researcher who, among the participating researchers, generates the output of the highest quality. Our paper allows for second, third, …, rth prizes, to be awarded to the contestants with, respectively, the second-, third-, …, rth highest qualities of output.

We show that the resulting multi-prize research tournament is strategically equivalent to a sequential multi-prize Tullock contest. This extends the "duality" between Tullock contests and research tournaments that previous work has established for single-prize settings: the winning probability in such tournaments is determined by the standard Tullock success function (Baye and Hoppe, 2003). With multiple prizes, the noisy ranking of a tournament is in fact equivalent to the sequential multi-prize Tullock contest, as applied, e.g., by Clark and Riis (1996, 1998), Szymanski and Valletti (2005) or Schweinzer and Segev (2012): such a contest awards a

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[1] This approach goes back to Arrow (1969) and Evenson and Kislev (1976) who suggest to view research as an experiment composed of several trials, each resulting in an observation, and taking the best observation in the sample as the outcome of the experiment.

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first prize to the best performer, eliminates this contestant from the pool of contestants and awards the second prize to the best performer among the remaining agents etc. The simultaneous-move research tournament and this nested sequence of Tullock contests give rise to the same multi-prize contest success function. Hence, their strategic equivalence.[2]

This finding, first, provides a micro-founded generalization of the simultaneous Tullock contest to multiple prizes. More importantly, it allows us to apply results on the optimal design of nested contests to multi-prize research tournaments. Notably, the general suboptimality of multiple prizes shown by Schweinzer and Segev (2012) carries over to tournaments: multiple prizes are inferior to single prizes, and it is optimal for the designer of a research tournament who wishes to incentivize as much research effort as possible to devote his entire prize money to a single, "large" prize.

2. The Fullerton–McAfee research tournament

2.1. Single prize

In the tournament framework by Fullerton and McAfee (1999) there is a fixed set $N = \{1, \ldots, n\}$ of $n > 1$ contestants who compete to win a single prize. Contestants spend efforts z_i (with $i = 1, \ldots, n$) that result in a random output quality x, distributed by the cumulative distribution function $F_i = F^{z_i}(x)$ where $F(x)$ has support on some closed interval $[0, \bar{x}]$. Output quality is independently distributed across contestants. Denoting the density of F by f, the density of contestant i's quality of output is given by

$$dF_i/dx = z_i F^{z_i - 1}(x) f(x).$$

The prize will be awarded to the contestant with the highest quality of output. Denote by p_1^i the probability that contestant i will be the winner, given effort levels $z = (z_i, z_{-i})$. Fullerton and McAfee (1999) as well as Baye and Hoppe (2003) have pointed out that in this setting a simple Tullock contest emerges, i.e., a contest with success function (CSF)

$$p_1^i(z_i, z_{-i}) = \frac{z_i}{Z}, \tag{1}$$

where $Z := \sum_j z_j$ denotes total effort. To see this, note that

$$\begin{aligned} p_1^i(z_i, z_{-i}) &= \int_0^{\bar{x}} \left(\prod_{j \neq i} F^{z_j}(x) \right) \cdot z_i F^{z_i - 1}(x) f(x) dx \\ &= z_i \int_0^{\bar{x}} F^{Z-1}(x) f(x) dx = z_i \int_0^1 u^{Z-1} du = z_i / Z. \end{aligned}$$

2.2. Multiple prizes

Consider the Fullerton–McAfee framework, but assume that there are $m > 1$ prizes (with fewer prizes than contestants; $m < n$). As in Moldovanu and Sela (2001), the values of the prizes are (weakly) decreasing from first to mth prize. Prizes are simultaneously awarded according to the contestants' outputs: the first, second, third, …, rth prize is allocated to the contestant with, respectively, the highest, second-, third-, …, rth highest quality of output. Denote the probability that contestant i wins the rth prize by $p_r^i(z_i, z_{-i})$.

[2] Fu and Lu (2012) derive a distribution-based equivalence between a multi-prize noisy-ranking contest model and a multi-prize nested contest model in which each contestant is ranked against others based on his/her favorable performance among multiple independent attempts, similar as in the Fullerton–McAfee research tournament.

Let us first focus on the second prize. The probability for contestant i to win this prize is equal to the probability that his effort z_i leads to the second-largest x. Defining $Z_{-j} := \sum_{k \neq j} z_k$, we obtain:

$$p_2^i(z_i, z_{-i}) = \frac{z_i}{Z} \sum_{j \neq i} \frac{z_j}{Z_{-j}}. \tag{2}$$

To see this, note that

$$\begin{aligned} p_2^i(z_i, z_{-i}) &= \int_0^{\bar{x}} \left(\sum_{j \neq i} (1 - F_j(x)) \cdot \prod_{t \neq i,j} F_t(x) \right) dF_i \\ &= \int_0^{\bar{x}} \left(\sum_{j \neq i} (1 - F^{z_j}(x)) \cdot F^{\sum_{t \neq i,j} z_t}(x) \right) \cdot z_i F^{z_i - 1}(x) f(x) dx \\ &= z_i \int_0^{\bar{x}} \left(-(n-1) F^{Z-1}(x) + \sum_{j \neq i} F^{Z_{-j} - 1}(x) \right) f(x) dx \\ &= z_i \int_0^1 \left(-(n-1) u^{Z-1} + \sum_{j \neq i} u^{Z_{-j} - 1} \right) du \\ &= z_i \left(-\frac{n-1}{Z} + \sum_{j \neq i} Z_{-j}^{-1} \right) = \frac{z_i}{Z} \cdot \sum_{j \neq i} \frac{z_j}{Z_{-j}}. \end{aligned}$$

Similar procedures can be applied to determine the probability to win a later prize. In particular,

$$p_r^i(z_i, z_{-i}) = \int_0^{\bar{x}} \sum_{N \setminus \{i\}} \left(\prod_{(r-1)\,\text{factors}} (1 - F_j(x)) \prod_{(n-r)\,\text{factors}} F_k(x) \right) dF_i(x),$$

where the summation "index" is meant to cover all permutations of $N \setminus \{i\}$. Applying this to the third prize, for example, we obtain:

$$p_3^i(z_i, z_{-i}) = \frac{z_i}{Z} \cdot \sum_{j \neq i} \left(\frac{z_j}{Z_{-j}} \cdot \sum_{k \neq i,j} \frac{z_k}{Z_{-\{i,j\}}} \right).$$

3. Results

Towards our analysis of the optimal allocation of prizes in research tournaments, we first establish a relationship between the simultaneous-move research tournament and nested multi-prize Tullock contests. In nested contests, the same CSF is applied recursively, i.e., the winner of the rth prize is determined by applying the CSF to the set of players without the winners of the first $r - 1$ prizes. Assuming identical players and a Tullock CSF, such contests are discussed, e.g., in Clark and Riis (1998) or Schweinzer and Segev (2012). The latter authors show that, in a nested contest, the probability for winning the rth prize can be obtained by "telescoping in" on the probabilities of winning the first prize in groups of diminishing size:

$$p_r^i(z_i, z_{-i}) = \sum_{N \setminus \{i\}} \left(p_1^i(z_i, z_{-i}^{n-1}) \cdot p_1^i(z_i, z_{-i}^{n-2}) \cdot \cdots \cdot p_1^i(z_i, z_{-i}^{n-r+1}) \right), \tag{3}$$

where z_{-i}^k denotes a vector of z_j with dimension k that does not include contestant i.

Proposition 1. *A simultaneous Fullerton–McAfee research tournament with m prizes ($1 \leq m < n$) is equivalent to a nested Tullock contest with the same m prizes.*

Proof. Consider the second prize. From (2),

$$p_2^i(z_i, z_{-i}) = \frac{z_i}{Z}\sum_{j\neq i}\frac{z_j}{Z_{-j}} = \sum_{j\neq i}\left(\frac{z_j}{Z}\frac{z_i}{Z_{-j}}\right)$$
$$= \sum_{N\setminus\{i\}}\left(p_1^j(z_j, z_{-j})\cdot p_1^i(z_i, z_{-i}^{n-2})\right).$$

For $r = 2$, this is the CSF (3) for a nested contest where p_1^i is the simple Tullock CSF (adjusted to the number of contestants). For $r > 2$, the claim follows analogously, conditional on $r - 1$ prizes having already been awarded. Then use an induction-type argument. ■

Schweinzer and Segev (2012) show for symmetric contestants that the optimal prize structure of a contest with the nested Tullock success function assigns the entire prize pool to the best-performing contestant, provided that a symmetric pure strategy equilibrium exists in the contest game. Given our result that Fullerton–McAfee tournaments are strategically equivalent to ne sted Tullock contests, the optimality of a single prize also holds for tournaments:

Corollary 2. *In the symmetric Fullerton–McAfee research tournament it is optimal for a designer who wishes to maximize total research efforts to allocate a given prize sum to a single prize rather than to several small prizes.*

As Schweinzer and Segev (2012) note for nested Tullock contests, multiple prizes dampen incentives to exert effort, compared to a single prize. The same then holds for the symmetric research tournaments due to Fullerton and McAfee (1999).

Clearly, it would be interesting to explore whether the equivalence between research tournaments and nested contests, and the superiority of single prizes, also holds for asymmetric players and/or asymmetric equilibria. We leave this question for future research.

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