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RESEARCH ARTICLE

Price competition in markets with customer testing: the captive customer effect

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Abstract We introduce product differentiation into the analysis of price competition in markets where suppliers test customers in order to assess whether they will pay for received goods or services. We find that, if the degree of differentiation is sufficiently high, suppliers may improve the average probability that their clientele will pay by charging higher prices. This helps suppliers to sustain high prices in equilibrium. Moreover, endogenizing locations in product space, we demonstrate that the high price level can be implemented in a pure-strategy subgame-perfect equilibrium with a high degree of differentiation. This is in contrast to the original Hotelling model with linear travel costs where a pure-strategy subgame-perfect equilibrium fails to exist.

Keywords Hotelling · Price competition · Testing · Mixed strategies · Iterated elimination of strictly dominated strategies

JEL Classification Numbers D83 · G21 · L13

1 Introduction

In a wide range of markets suppliers face uncertainty about the profitability of serving customers. This includes markets where suppliers provide a product or service without knowing whether the customer will eventually pay for it. Examples are credit, venture

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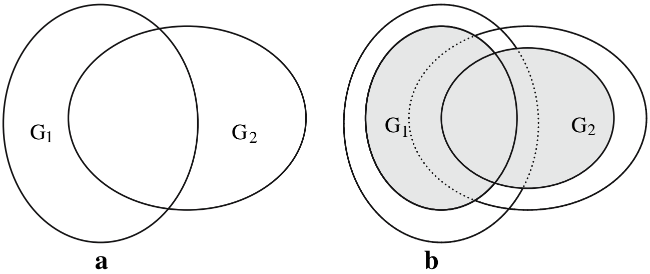

Fig. 1 Price competition with customer testing

capital, investment banking, labor, and consulting markets. In other markets, suppliers contract with customers without knowing whether these customers will impose additional costs on them. Examples include insurance and housing markets.

A typical feature of these markets is that suppliers test their customers by reviewing proposed projects, performing tryouts, or conducting interviews. Such tests are usually informative, but not perfect. Thus, in the case of two suppliers, there may be customers who fail the test of one supplier and are *captive* for the supplier whose test they have passed, and customers with favorable test results from both suppliers who are thus *non-captive* for any of the suppliers. If suppliers use their tests independently of each other, they will, however, not be able to distinguish whether a customer who passed the own test is a captive or a non-captive customer. The problem is that captive customers will impose losses with higher probability (since they passed only supplier's test) and may, on average, be even *detrimental* to the supplier.[1]

The main goal of our paper is to examine how the presence of such captive customers affects price competition in a duopoly model with differentiated products. Previous research on price competition with homogenous products and different customer groups (cf. Varian 1980; Broecker 1990) suggests that the presence of captive customers (whether beneficial or detrimental) typically leads to a Bertrand–Edgeworth type of competition such that no price equilibria in pure strategies exist. The situation with customer testing is illustrated in Fig. 1a: supplier 1(2) makes offers only to customers who pass the own test, G_1 (G_2), where $G_1 \backslash G_2$ are captive customers for supplier 1 (and $G_2 \backslash G_1$ for supplier 2), while both suppliers compete for customers who passed both's test, $G_1 \cap G_2$. For simplicity, suppose that the suppliers charge the same price and both get half the customers in $G_1 \cap G_2$. If, say, supplier 1 reduces his price below 2's price, then without product differentiation, supplier 1 will attract everyone in $G_1 \cap G_2$, that is, all non-captive customers. Thus, by undercutting the rival's price, a supplier can improve his customer base, which precludes the existence of a pure-strategy equilibrium.

The present paper shows that with product differentiation, the situation is different: suppose that supplier 1 is located in the center of oval G_1 and distance matters such that for a given price supplier 1 serves only customers in the inner oval (as illustrated in Fig. 1b), and both suppliers compete for customers in the intersection of the two

[1] That is, the expected profit from serving a captive customer may be negative.

shaded ovals (this does not necessarily describe an equilibrium). It will be shown that in this situation price changes have two effects: when a supplier lowers its price, the relevant shaded oval will expand. This supplier thus attracts further non-captive customers (from the set $G_1 \cap G_2$), but it will also attract further captive customers (from the set $G_1 \backslash G_2$) who, for a higher price, would be left out. Likewise, by increasing the price, the supplier may lose some non-captive customers, but also deter some captive ones. In particular, we find that captive customers react *more sensitively* to suppliers' price changes than non-captive customers (the *captive customer effect*). Intuitively, in order to attract an additional *non-captive customer* in the product/service space, a supplier's price reduction has to compensate that customer for both, the fact that the own product/service characteristics do not meet the preferences of the customer perfectly and also the fact that the rival's product/service characteristics meet the preferences of the customer better. On the other hand, in order to attract an additional *captive customer* , the supplier has to compensate that customer only for the first kind of utility loss. As a consequence, a *higher* price can *improve* the composition of the supplier's clientele by removing relatively more captive than non-captive customers.

Empirical observations suggest that product differentiation is indeed a pervasive phenomenon in markets with customer testing. Banks, for instance, as Allen and Gale (2000) observe, "offer different menus of accounts, or concentrate on different types of lending business; they attract a different mix of retail or wholesale funds; they may diversify into nonbank products such as insurance or mutual funds. [...] Differences in size and products and specialized knowledge make them imperfect substitutes." (p. 235). Similarly, investment firms typically differ in their equity and advisory work as well as their geographical scope of processing transactions.[2]

In this paper, we seek to analyze the situation illustrated in Fig. 1b more formally. For this we combine, in Sect. 2, the standard Hotelling (1929) approach where two suppliers are located on the unit interval of customer tastes with a simple customer testing model in which suppliers test all potential customers and then compete a là Bertrand by offering contracts only to those customer who passed the own test, as in Broecker (1990). Broecker has shown that there exists no pure-strategy equilibrium in his model due to the incentive to undercut the rival's price and thereby attract all non-captive customers.

In Sect. 3, we show that the dimension of product differentiation restores the possibility of smooth demand effects for non-captive customers and hence existence of pure-strategy price equilibrium. But, more importantly, we find that the game possesses two different types of pure-strategy equilibria for a sufficiently high degree of differentiation. In the first, prices are "low" so that all captive customers will be served. In the other, more interesting equilibrium, prices are "high" so that not all captive customers will be served. This equilibrium is supported by the *captive customer effect*, that is, the high price is used to fend off relatively more captive than non-captive customers. Note that this high-price equilibrium has no analog in the standard Hotelling model, which admits a pure-strategy equilibrium, but only of the low-price type.

[2] See, e.g.,"Strategies for corporate and institutional banking", *The Economist*, April 4, 2002. For recent empirical studies of product differentiation in financial markets, see e.g. Cohen and Mazzeo (2004) and Kim et al. (2005).

In Sect. 4, we investigate whether the high-price equilibrium may be implemented in a subgame-perfect equilibrium of the two-stage (location-then-price) extension of our model. Endogenizing the suppliers' locations in product space, we are able to demonstrate for a subset of parameters that both, a high degree of product differentiation and a high level of prices are in fact implemented in a pure-strategy subgame-perfect equilibrium of the two-stage extension of our model. Interestingly, the equilibrium is sustained by the existence of the additional low-price equilibrium, which works as a threat to punish any reduction in the degree of differentiation. By contrast, in the original Hotelling model with linear travel costs, suppliers wish to reduce the degree of differentiation as long as there exists a pure-strategy price equilibrium for the corresponding subgames. This leads them into a region where the only price equilibria are in mixed strategies. Our analysis shows that in the presence of detrimental captive customers, there is no such general tendency for suppliers to be drawn into the region where a pure-strategy price equilibrium ceases to exist.

We provide some concluding remarks in Sect. 5.

The analysis in our paper contributes to two literatures. First, our model introduces product differentiation into models of price competition in markets where suppliers assess whether potential customers will pay for received goods or services. Such models of customer testing without product differentiation are considered by Broecker (1990), Riordan (1993), and Taylor (2004). Riordan considers a model similar to that of Broecker, but assumes that signals are continuous rather than discrete. Taylor analyzes a situation where suppliers first post prices and then test their applicants, where the information acquisition levels are endogenously determined.

Second, the model adds customer testing to Hotelling location models. The existing literature offers several variants of the Hotelling model. However, to the best of our knowledge, none of them considers markets where suppliers test customers in order to assess whether they will pay for received goods or services. In addition, our paper seems to be the first to consider a Hotelling-type model where a pure-strategy price equilibrium may be implemented as subgame-perfect equilibrium of the two-stage (location-then-price) game, although there are subgames that do not have a pure-strategy price equilibrium. Thus, analyzing the first-stage location decisions requires determining the profitability of deviations to locations that involve no pure-strategy price equilibrium. By contrast, in previous variants of the Hotelling model the possible non-existence of pure-strategy price equilibria has been explicitly ruled out for every location pair [see, for instance, d'Aspremont et al. (1979), de Palma et al. (1985), Anderson et al. (1992), and Anderson et al. (1997)].[3] Unfortunately, the analysis of mixed-strategy equilibria in our framework is very difficult. In fact, even for the standard Hotelling model with linear travel costs it seems impossible to analytically derive the mixed-strategy equilibrium profits for every subgame.[4] Our approach to the study of subgame-perfect equilibria differs from the usual one by

[3] These models assume quadratic travel costs instead of linear or use alternative demand formulations.

[4] Approximate equilibria for the Hotelling model with linear travel costs are derived by Osborne and Pitchik (1987) using computational methods. The paper by Moscarini and Ottaviani (2001) appears to be the latest attempt to tackle the problem of mixed-strategy price equilibria in a variant of the Hotelling game where consumers have private information about their preferences.

not requiring a derivation of the mixed-strategy equilibrium profits for subgames that do not possess a pure-strategy equilibrium. The idea is to make use of the fact that only serially undominated actions[5] can be played with positive probability in a mixed-strategy equilibrium, which enables us to obtain lower and upper bounds for the equilibrium distributions in subgames that do not possess pure-strategy equilibria.[6] These bounds are then used to derive an upper bound for the mixed-strategy equilibrium payoffs.

2 Model

There are two suppliers, $i = 1, 2$, who offer contracts to provide a costly product or service. The costs of providing the product or service is normalized to 1. The suppliers are differentiated à la Hotelling. There is a continuum of customers, uniformly located on the segment $[0, 1]$.[7] Each customer has a reservation value of $Y > 1$ for the product or service, and anticipates that it incurs a disutility of $t \geq 0$ per unit of distance between its own location and the location of the supplier with whom the contract is concluded. Each supplier i's contract stipulates a price for the product or service, p_i. Customers contract with one or the other supplier. We assume that customers have an outside option that pays zero, so they will accept a contract only if their utility is non-negative.

Each customer is either a good type (type g) or a bad type (type b). A good type will be able to pay for sure for received products and services. A bad type believes that he will be able to pay for sure, but ends up paying nothing. Suppliers clearly want to avoid trading with the "nuisance" (type b) customers, but they cannot observe whether they are dealing with a nuisance or a "genuine" (type g) customer. Types are unknown to suppliers as well as customers. Hence, as in Broecker (1990), signaling is not an issue. Instead, suppliers screen all customers by conducting tests. Tests are costless and stochastically independent for each customer. Each test provides a private signal, $S \in \{B, G\}$, about the customer's type, where B denotes an unfavorable signal and G a favorable one. The suppliers' prior belief of type b is λ, where $0 < \lambda < 1$. Suppliers update their prior beliefs according to Bayes' rule.

Let $q(S|s)$ denote the conditional probability of observing signal S, given an s-type customer. Assume that $0 \leq q(B|g) < q(B|b) \leq 1$, i.e., the test statistic is potentially informative. Note that $q(G|g) = 1 - q(B|g)$ and $q(G|b) = 1 - q(B|b)$. To ease the exposition, we make use of the following definitions:

$$q_1 := q(g|G, B) = \frac{1}{q_4}(1-\lambda)\, q(G|g)\, q(B|g)$$

[5] This is the set of actions that survives iterated elimination of stricly dominated actions.

[6] In contrast to the approach by Kreps and Scheinkman (1983), the bounds need not be elements of the support of the equilibrium distribution.

[7] As will be seen below, the *captive customers effect* identified in this paper prevails also under a more general customer distribution.

$$q_2 := q\,(g|G,G) = \frac{1}{q_3}\,(1-\lambda)\,(q\,(G|g))^2$$
$$q_3 := q\,(G,G) = \lambda\,(q\,(G|b))^2 + (1-\lambda)\,(q\,(G|g))^2$$
$$q_4 := q\,(G,B) = \lambda\,q\,(G|b)\,q\,(B|b) + (1-\lambda)\,q\,(G|g)\,q\,(B|g)$$

That is, q_4 is the proportion of customers that have passed one supplier's test, but failed that of the other supplier, i.e., *captive customers* of the supplier whose test they have passed. Similarly, q_3 is the proportion of customers that passed both suppliers' tests, i.e., *non-captive customers*. q_1 is a supplier's posterior belief that a customer is of type g, given the customer belongs to the group of captive customers, while q_2 is a supplier's posterior belief that a customer is of type g, given that the customer belongs to the group of non-captive customers.

Customers who got an offer from one or both suppliers decide whether to accept at most one of them. Suppliers do not share any information and select their customers solely on the basis of their own test. Quite naturally, it is assumed that the expected payment from a customer who fails a test is too low to make an offer to this customer attractive.[8] Furthermore, offers cannot be made contingent on the location of customers.[9] Thus, each supplier i's strategy consists of a contract offer p_i to customers who passed the own test.

The model has the following two polar cases, depending on the informativeness of the test statistic:

1. $0 < q\,(B|g) < q\,(B|b) = 1$, which gives $q_1 = q_2 = 1, 0 < q_3 < 1, 0 < q_4 < 1$. In this case, a supplier correctly infers from a G-signal that the customer will pay for sure.
2. $0 = q\,(B|g) < q\,(B|b) < 1$, which gives $q_1 = 0, 0 < q_2 < 1, 0 < q_3 < 1, 0 < q_4 < 1$. In this case, a supplier correctly infers from a B-signal that the customer will not be able to pay. On the other hand, a G-signal does *not* guarantee that the customer will pay for sure.

Note that the first case yields a setting which is equivalent to the standard Hotelling model where suppliers face no uncertainty about the profitability of serving customers. In contrast, the second case describes a situation where serving a customer who passed the own test may not be profitable.

In order to highlight the effect of *detrimental captive customers* and to facilitate the analysis, we will henceforth restrict attention to the second polar case. Note that in more realistic settings with $0 < q\,(B|g) < q\,(B|b) < 1$, we would have $q_1 > 0$, i.e., the suppliers' posterior belief that a captive customer (who passed only one test) is a good (type g) customer would be strictly positive. Similarly for customers' beliefs, letting μ denote the customers' prior belief of being type b, customers who passed

[8] Note that otherwise suppliers would have no incentive for customer testing in the first place.

[9] This assumption seems rather natural in the context of non-geographic product differentiation, where preferences are typically unobservable. Furthermore, in a geographic context, the assumption seems justified when customers are able to conceal their location, for example, by using a different correspondence address. Note that customers would indeed have an incentive to do so if offers would depend on locational aspects.

only one test and update any prior belief of $0 < \mu < 1$ according to Bayes' rule, would believe to be type g with strictly positive probability. Hence, the captive customers' demand would be price-sensitive. Such a setting would, however, complicate the notation and analysis considerably. Therefore, for tractability, we restrict attention to the second polar case ($0 = q(B|g) < q(B|b) < 1$) and ensure price-sensitive demands of captive customers in this case by assuming that each customer initially believes that he will pay for sure ($\mu = 0$).[10] In fact, the assumption of strong customers' priors seems not too unrealistic in the context we wish to study. For financial markets, DeBondt and Thaler (1995, p. 389), for instance, report that "perhaps the most robust finding in the psychology of judgement is that people are overconfident", that is, people are unrealistically optimistic about their ability, power, and the precision of their knowledge. Nevertheless, we would like to emphasize that, in the alternative setting with $0 < q(B|g) < q(B|b) < 1$ and $0 < \mu < 1$, the *captive customer effect* identified in this paper would continue to play essentially the same role for the existence of the different types of pure-strategy price equilibria, as long as the suppliers' posterior belief q_1 that captive customers will pay for received goods and services is not too high.[11] In other words, while the presence of price-sensitive captive customers will turn out to be critical in determining the qualitative properties of the model—and assuming $q(B|g) = 0$ (i.e. the second polar case) and $\mu = 0$ generates such customers—this sort of "bounded rationality" will not be critical per se in our model.

3 Equilibrium analysis

Each supplier serves two classes of customers: captive customers and non-captive customers. Let z_1 denote the distance of supplier 1's location from 0, and z_2 the distance of supplier 2's location from 1, where $z_1 + z_2 \leq 1$.

A customer who passed both suppliers' tests and is indifferent between contracting with supplier 1 and contracting with supplier 2 is located at $\hat{x}$, where $\hat{x}$ is uniquely determined by $Y - p_1 - t\left(\hat{x} - z_1\right) = Y - p_2 - t\left(1 - z_2 - \hat{x}\right)$. Hence, $\hat{x} = \frac{1}{2}(1 - z_2 + z_1) + \frac{1}{2t}(p_2 - p_1)$, and supplier 1's demand from non-captive customer is given by

$$x = \begin{cases} 1 & \text{if } \ p_2 - p_1 > t(1 - z_2 - z_1) \\ \frac{1}{2}(1 - z_2 + z_1) + \frac{1}{2t}(p_2 - p_1) & \text{if } \ |p_2 - p_1| \leq t(1 - z_2 - z_1) \\ 0 & \text{if } \ p_1 - p_2 > t(1 - z_2 - z_1) \end{cases} \tag{1}$$

while supplier 2's demand from non-captive customers is given by $1 - x$, respectively.

[10] Note that in this case the customers' beliefs are not affected by the test outcomes under Bayes' rule. Hence, a captive customer's demand remains price-sensitive.

[11] As q_1 goes to 1, the model converges to the first polar case, in which a favorable signal guarantees that the customer will pay for sure. For the case of $0 < q(B|g) < q(B|b) < 1$ and $0 < \mu = \lambda < 1$, i.e., when customers and suppliers have the same priors, we have also verified, by means of numerical examples, that the nature of the results continues to be essentially the same as in Sect. 3 of our paper.

A customer who passed supplier 1's test and failed supplier 2's test and is indifferent between contracting with supplier 1 and not contracting at all is located at $\hat{x}_1$, where $\hat{x}_1$ is uniquely determined by $Y - p_1 - t\left(\hat{x}_1 - z_1\right) = 0$. Hence, $\hat{x}_1 = z_1 + \frac{1}{t}\left(Y - p_1\right)$, and supplier 1's demand from captive customers is given by

$$x_1 = \min\left(z_1 + \frac{1}{t}\left(Y - p_1\right), 1\right) \tag{2}$$

Similarly, supplier 2's demand from captive customers is given by

$$1 - x_2 = 1 - \max\left(0, 1 - z_2 - \frac{1}{t}\left(Y - p_2\right)\right) \tag{3}$$

Recall that the proportion of captive customers is given by q_4 and the proportion of non-captive customers by q_3. For $q_1 = 0$, $q_2 > 0$, suppliers' duopoly profits are therefore given by

$$\Pi_1 = -q_4 x_1 + q_3\left(p_1 q_2 - 1\right) x \tag{4}$$

$$\Pi_2 = -q_4(1 - x_2) + q_3\left(p_2 q_2 - 1\right)(1 - x), \tag{5}$$

where x_1, x_2, and x are given by (2), (3), and (1), respectively. The first term of the profit functions represents the expected loss from contracting with captive customers,[12] and the second term gives the expected profit from contracting with non-captive customers.

Existence of pure-strategy equilibria

In the following we examine the existence and properties of symmetric pure-strategy equilibria. There are two possible cases to distinguish: $0 < x_2 < x < x_1 < 1$ and $0 = x_2 < x < x_1 = 1$. In the second case, there is market coverage with respect to captive customers. In the first case, there is no market coverage with respect to captive customers. This case is illustrated in Fig. 2, where we plot customers' net utility from contracting with supplier 1 and 2 for two different price choices of supplier 1, $p_1 > p_1'$.

[12] The probability that a captive customer is of type g is zero, i.e., $q_1 = 0$, whereas the the cost of providing the service or product is 1.

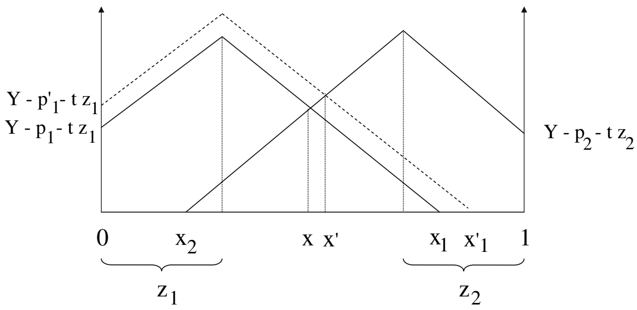

Fig. 2 No market coverage with respect to captive customers

Suppose now that the solution of suppliers' maximization yields a pure-strategy equilibrium with $0 < x_2 < x < x_1 < 1$, i.e., no market coverage with respect to captive customers. The equilibrium prices are then of Type I:[13]

$$p_1^{nc} := t + \frac{1}{3}t\,(z_1 - z_2) + \frac{1}{q_2} + \frac{2}{q_3 q_2}q_4 \tag{6}$$

$$p_2^{nc} := t - \frac{1}{3}t\,(z_1 - z_2) + \frac{1}{q_2} + \frac{2}{q_3 q_2}q_4, \tag{7}$$

Note that in this case an increase in the relative proportion of captive customers, q_4/q_3, raises the price of each supplier. This in turn makes accepting a contract offer unattractive for the most distant captive customers, and simultaneously increases each supplier's revenue from contracts with non-captive customers.[14]

Suppose next that the solution of suppliers' maximization yields a pure-strategy equilibrium with $0 = x_2 < x < x_1 = 1$, i.e., market coverage with respect to captive customers. The equilibrium prices are then of Type II:[15]

$$p_1^{c} := p_1^{nc} - \frac{2}{q_3 q_2}q_4 \tag{8}$$

$$p_2^{c} := p_2^{nc} - \frac{2}{q_3 q_2}q_4 \tag{9}$$

Substituting (6) and (7) into (8) and (9) reveals that in this case an increase in the relative proportion of captive customers, q_4/q_3, leaves the price of each supplier unaffected.

Let $\Omega^Y = \{Y \in \mathbf{R}_+ : Y \geq 1\}$ and $\Omega^t = \{t \in \mathbf{R}_+ : t \geq 0\}$. In the parameter space $\Omega^Y \times \Omega^t$, we seek a region A^I such that the game possesses a pure-strategy equi-

[13] The derivation is heuristic at this point—the existence of the equilibrium of Type I is established in Proposition 1 below and proved in Appendix A.

[14] As in the Hotelling model, each supplier's price is also increasing in travel costs, t, the degree of differentiation, $|z_1 - z_2|$, and average costs, $1/q_2$.

[15] The existence of the equilibrium of Type II is established in Proposition 1 below. See Appendix A for the proof.

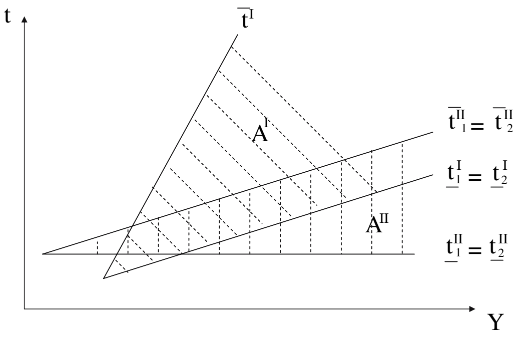

Fig. 3 Regions A' and A'' for the case of $z_1 = z_2 = 0$

librium of Type I. As the following proposition shows, for z_1, z_2 small enough, this region can be characterized by

$$A^{I} := \left\{(t, Y) : \max\left(\underline{t}_1^{I}, \underline{t}_2^{I}\right) \leq t \leq \bar{t}^{I}\right\} \tag{10}$$

where

$$\underline{t}_1^{I} := \frac{3}{2}\frac{2Yq_3q_2-2q_3-3q_4}{q_3q_2(6-2z_1-z_2)}, \quad \underline{t}_2^{I} := \frac{3}{2}\frac{2Yq_3q_2-2q_3-3q_4}{q_3q_2(6-2z_2-z_1)} \tag{11}$$

$$\bar{t}^{I} := 2\frac{Yq_3q_2-2q_4-q_3}{q_3q_2(3-z_2-z_1)} \tag{12}$$

If travel costs are too low, i.e. $t < \max\left(\underline{t}_1^{I}, \underline{t}_2^{I}\right)$, then a price choice of $\left(p_1^{nc}, p_2^{nc}\right)$ leads to a situation where $0 < x_2$ and $x_1 < 1$ does not hold, i.e., there is market coverage with respect to captive customers. On the other hand, if travel costs are too high, i.e., $t > \bar{t}^{I}$, prices $\left(p_1^{nc}, p_2^{nc}\right)$ are not consistent with $x_1 > x$ and $x_2 < x$, i.e., there are local monopolies with respect to non-captive customers.

Furthermore, for z_1, z_2 small enough, there exists a region A^{II} in $\Omega^{Y} \times \Omega^{t}$, in which the game has a pure-strategy equilibrium of Type II. This region is characterized by

$$A^{II} := \left\{(t, Y) : \max(\underline{t}_1^{II}, \underline{t}_2^{II}) \leq t \leq \min(\bar{t}_1^{II}, \bar{t}_2^{II})\right\} \tag{13}$$

where

$$\begin{aligned} \underline{t}_1^{II} &:= \frac{18q_4}{q_3q_2(z_1+3-z_2)^2}, \quad & \underline{t}_2^{II} &:= \frac{18q_4}{q_3q_2(z_2+3-z_1)^2} \\ \bar{t}_1^{II} &:= \frac{3}{2}\frac{2Yq_3q_2-q_4-2q_3}{q_3q_2(6-2z_1-z_2)}, \quad & \bar{t}_2^{II} &:= \frac{3}{2}\frac{2Yq_3q_2-q_4-2q_3}{q_3q_2(6-2z_2-z_1)} \end{aligned} \tag{14}$$

In this case, if travel costs are too low, i.e. $t < \max(\underline{t}_1^{II}, \underline{t}_2^{II})$, then the profits of at least one supplier associated with a prices $\left(p_1^{c}, p_2^{c}\right)$ fall below zero. If travel costs are too high, i.e. $t > \min(\bar{t}_1^{II}, \bar{t}_2^{II})$, then prices $\left(p_1^{c}, p_2^{c}\right)$ lead to a situation where $0 = x_2$ and $x_1 = 1$ does not hold, i.e., the market is not covered with respect to each supplier's captive customers.

Figure 3 depicts regions A^{I} and A^{II} for the case of $z_1 = z_2 = 0$.

As z_1 and/or z_2 are increased from 0, one can show that any pure-strategy equilibrium eventually ceases to exist, similarly as in the original Hotelling model with linear travel costs. For a sufficiently small degree of differentiation, suppliers choose, with positive probability, to undercut each other so as to win the customer located exactly at the rival's location and due to linear travel costs also all customers located in the rival's hinterland.[16] Moreover, in our setting, suppliers choose with positive probability to effectively exit the market when the degree of differentiation is sufficiently small, similarly as in the Broecker model.

The next proposition summarizes the results. The proof is placed in Appendix A.

Proposition 1 *The game has at least one (possibly mixed) equilibrium for any* z_1, z_2*. The following statements hold for* z_1, z_2 *small enough:*

1. *There exists a pure-strategy equilibrium of Type I in region* A^I.
2. *There exists a pure-strategy equilibrium of Type II in region* A^{II}.
3. *The equilibrium of Type I involves a higher profit for each supplier than the equilibrium of Type II in region* $A^I \cap A^{II}$.
4. *There exists no other pure-strategy equilibria in region* $A^I \cup A^{II}$.

The proposition establishes existence of the two different types of symmetric pure-strategy equilibria, i.e., the high-price equilibrium of Type I (without market coverage with respect to captive customers) and the low-price equilibrium of Type II (with market coverage) if the degree of differentiation is sufficiently high. This is in contrast to the results for the Broecker model, where a pure-strategy equilibrium fails to exist, and also in contrast to the results for the Hotelling model, where any pure-strategy equilibrium, if it exists, is of the low-price type.

Previous research on price competition with detrimental captive customers (Broecker 1990) has shown that without product differentiation, a supplier can improve its customer base by undercutting the price of the rival supplier. Proposition 1 reveals that a sufficiently high degree of product differentiation gives rise to a new effect, which we call the *captive customer effect*: for $x_1 < 1$, $x_2 > 0$, we have

$$\frac{\partial\,(x/x_1)}{\partial p_1} = -\frac{1}{2}\frac{Y - t + z_2 t - p_2}{(z_1 t + Y - p_1)^2} > 0 \tag{15}$$

since $x_2 > 0 \Leftrightarrow Y - t + z_2 t - p_2 < 0$ in the high-price equilibrium. That is, the market boundary with respect to supplier 1's captive customers, x_1, reacts *more sensitively* to a price change by supplier 1 than the market boundary with respect to non-captive customers, x (see Fig. 2 above); and similarly for supplier 2. This *captive customer effect* might seem surprising at first sight. Note, however, that in order to attract an additional non-captive customer, the supplier's price reduction has to compensate that customer for both, first, the fact that the own product characteristics do not meet the preferences of the customer perfectly, and second, the fact that the rival's product characteristics meet the preferences of the customer better. In contrast, an additional captive customer is already attracted once this customer is compensated for the first kind of utility loss.

[16] The mixed-strategy equilibria in the original Hotelling model have been analyzed by Osborne and Pitchik (1987).

The *captive customer effect* implies that, for a sufficiently high degree of differentiation, a supplier can *improve* the composition of its clientele by *increasing* its price from a high level, such as the Type I equilibrium level. As a result, high prices can be supported in equilibrium. On the other hand, when prices are low such that $x_1 = 1, x_2 = 0$ (i.e., markets for captive customers are covered), we have $\partial\left(\frac{x}{x_1}\right)/\partial p_1 = -\frac{1}{2t} < 0$. That is, a supplier can *improve* the composition of its clientele by *reducing* its price from a low level. Thus, low prices can also be supported in equilibrium.

It seems worth noting that this multiplicity of equilibria for a high degree of differentiation is not an artefact of the uniform customer distribution along the closed interval. Consider a more general distribution with the property that there is more weight at the center and less weight at the boundaries of the distribution. Note that the captive customer effect of charging a higher price is proportional to the height of the customer density at the market boundaries, x_1 and x_2, for supplier 1 and 2, respectively. Since higher prices imply that the market boundaries, x_1 and x_2, are closer to the center, high prices may form an equilibrium, because they would be 'supported' by an associated strong captive customer effect. On the other hand, since low prices imply that the market boundaries are further away from the center, low prices may form an equilibrium as well because they would be 'supported' by a weak captive customer effect.

4 Endogenizing the location decisions

It is well known that endogenizing the suppliers' location decisions in the original Hotelling model with linear travel costs yields no pure-strategy subgame-perfect equilibrium of the two-stage (location-then-price) game. That is, suppliers will always choose locations for which the only price equilibria are in mixed strategies. Is this also the case in our model? More precisely, is the *captive customer effect* identified above generally destroyed by the suppliers' location decisions?

As we demonstrate in the following, the location-then-price extension of our model admits subgame-perfect equilibria in pure strategies in which the high-price equilibrium is implemented. These equilibria involve a high degree of differentiation.[17] In order to deal with the technical complexity of the subgame-perfect equilibrium analysis, we will consider the location decisions only for a small subset of parameters. In a technical sense, we will prove that the set of parameters where the *captive customer effect* persists when suppliers can choose their locations is non-empty.

We will distinguish two concepts of subgame-perfect equilibria. One requires strategies to be Pareto perfect (cf. Bernheim et al. 1987; Fudenberg and Tirole 1991, p. 175). That is, suppliers are restricted not to play a Pareto-dominated equilibrium in any price game. The other involves cooperation-inducing strategies. That is, suppliers employ strategies that threaten to meet any deviation in the location-stage by playing

[17] As d'Aspremont et al. (1979) have shown, the introduction of quadratic travel costs in the Hotelling model yields a subgame-perfect equilibrium with maximal differentiation. For quadratic travel cost, Meagher and Zauner (2005) identify uncertainty about customer tastes as an additional differentiation force. The analysis in the present paper demonstrates that uncertainty about the profitability of serving customers and customer testing introduce an incentive for a high, possibly maximal degree of differentiation in the original Hotelling model with linear travel costs.

a Pareto-dominated equilibrium in the price game, provided that such an equilibrium exists (cf. Benoit and Krishna 1985; Vives 1999, p. 303).

The following propositions summarize our results. For the analysis and proofs, see Appendix B.

Proposition 2 *There exists a non-empty set of parameters such that the game has a unique pure-strategy Pareto-perfect equilibrium. This equilibrium involves high prices,* $p_1^{nc} = p_2^{nc}$*, in the price-setting stage.*

To prove the proposition we construct an equilibrium with locations at which the game possesses both types of price equilibria (high-price and low-price), but no high-price equilibrium if one of the suppliers would deviate by moving towards the center. Thus, in intuitive terms, suppliers are induced to "punish" any unilateral reduction in the degree of differentiation by switching from the high-price equilibrium to the low-price equilibrium, if it exists, or a mixed-strategy equilibrium otherwise. Checking the profitability of deviations that involve mixed-strategy equilibria turns out to be complicated. For such deviations, suppliers either have an incentive to lower the price in order to win over the customers located in the rival hinterland (resulting in mixed-strategy equilibria of the "Hotelling type") or to set the reservation price Y with positive probability and effectively stay away from the market (resulting in mixed-strategy equilibria of the "Broecker type"). The problem is that it seems impossible to analytically derive the mixed-strategy equilibria (similarly as for the original Hotelling model, see Osborne and Pitchik 1987). Moreover, it seems not even possible to determine the bounds of the support of the equilibrium distributions, ruling out the approach used in Kreps and Scheinkman (1983) or Moscarini and Ottaviani (2001). We confront this problem by making use of the fact that only serially undominated prices can be played with positive probability in a mixed-strategy equilibrium. This allows us to derive a sufficiently tight upper bound of the lowest price in the support of supplier 1's equilibrium distribution and a sufficiently tight upper bound of the support of supplier 2's equilibrium distribution, where both bounds need not be elements of the support. We use these bounds to derive an upper bound for the mixed-strategy equilibrium payoff of supplier 1, which is in turn used to show that deviations to locations where there exists no pure-strategy equilibrium are not profitable.

The next proposition applies the concept of cooperation-inducing strategies [as in Benoit and Krishna (1985), in their analysis of finitely repeated stage games] to establish existence of subgame-perfect equilibria in which the high-price equilibrium is implemented.

Proposition 3 *There exists a non-empty set of parameters such that the game has a subgame-perfect equilibrium in cooperation-inducing strategies. This equilibrium involves high prices,* $p_1^{nc} = p_2^{nc}$*, in the price-setting stage.*

Similarly as above, we construct an equilibrium which involves locations at which the game possesses both types of price equilibria (high-price and low-price). The idea behind the equilibrium of Proposition 3 is the following. If, say, supplier 1 does not cooperate in the location stage by deviating towards the center, it triggers a switch from the high-price equilibrium to the low-price equilibrium, even when both types of equilibria still exist under the deviation. That is, playing the less profitable low-price equilibrium is used as a credible threat in the sense of subgame perfection. Otherwise, the equilibrium involves switching from the high-price equilibrium to the low-price equilibrium, if it exists, or a mixed-strategy equilibrium if there exists no pure-strategy equilibrium.

Notice that our results differ sharply from the results of the original Hotelling model with linear travel costs, where a pure-strategy subgame-perfect equilibrium fails to exist. Since there is no multiplicity of pure-strategy price equilibria in the original Hotelling model, each supplier always benefits from unilaterally reducing the degree of differentiation as long as there exists a pure-strategy price equilibrium. Suppliers are therefore 'drawn' into the region where there exists no pure-strategy price equilibrium for the corresponding subgames (cf. d'Aspremont et al. 1979). As we have demonstrated, the presence of detrimental captive customers in our model can result in multiple price equilibria and thereby sustain existence of subgame-perfect equilibria in pure strategies. These equilibria involve a high degree of differentiation and high prices.

5 Concluding remarks

This paper has introduced the dimension of product differentiation into the analysis of price competition in markets where suppliers face uncertainty about the profitability of serving a customer and conduct tests in order to reduce this uncertainty. A priori, it seems that the presence of captive customers (who pass only one supplier's test and will therefore impose losses with a higher probability) will be detrimental to the suppliers. Our analysis reveals, however, that for a sufficiently high degree of differentiation the presence of such customers implies that suppliers can improve the composition of their clientele by charging *higher prices*.[18] The reason is that captive customers are more sensitive to the suppliers' decisions than non-captive customers. This *captive customer effect* helps suppliers to sustain a high-price equilibrium whenever products are sufficiently differentiated. Only a small fraction of captive customers is actually served in this equilibrium.

Moreover, endogenizing the suppliers' location decisions in product space, we demonstrate that our model in fact admits subgame-perfect equilibria where the degree of differentiation is high enough such that a pure-strategy high-price equilibrium is implemented. Hence we were able to demonstrate that, in contrast to the original Hotelling game with linear travel costs, there is no general tendency in our context that suppliers are drawn into the region where a pure-strategy price equilibrium ceases to exist. Similarly as for the Hotelling model, it is apparently impossible to analytically derive the mixed-strategy equilibria for location pairs where there exists no pure-strategy price equilibrium. We have confronted this problem by applying the technique of

[18] In fact it is possible to show that suppliers may be better-off in a setting with imperfect testing, as is considered in our model, than in a setting with perfect testing. Note that for perfect testing we have $q_2 = 1, q_3 = 1 - \lambda, q_4 = 0$. We leave the question whether this might lead to underinvestment in testing technology for future research.

iterated elimination of strictly dominated strategies. The approach might be of independent interest for the analysis of other multi-stage games that do not allow for an analytical solution of some subgames. Future research on price competition in markets with customer testing might investigate the presence and consequences of the *captive customer effect* in more general demand settings.

Appendix A

Proof of Proposition 1 Existence of at least one (possibly mixed) equilibrium for any z_1, z_2 can be established by using the concept of better-reply security of Reny (1999):

Theorem (Reny 1999)[19] *Let X_i and $u_i : X \to \mathbf{R}$ be player i's pure strategy set and payoff function, respectively, where $X = \times_{i=1}^N X_i$. Suppose that $G = (X_i, u_i)_{i=1}^N$ is a compact, Hausdorff game. Then G possesses a mixed strategy Nash equilibrium if its mixed extension, $\bar{G}$, is better-reply secure.*[20] *Moreover, $\bar{G}$ is better-reply secure if it is both reciprocally upper-semicontinuous and payoff secure.*

Note first that the relevant strategy sets are compact intervals of prices and a Hausdorff (indeed, metric) space. As shown in Reny, reciprocal upper semicontinuity holds if the sum of the players' payoffs is upper semicontinuous on the relevant set of pure strategies. Note that for $p_2 > t(1 - z_2 - z_1) + p_1$ all non-captive customers trade at price p_1, whereas at $p_2 = t(1-z_2-z_1)+p_1$, some non-captive customers trade at p_2. Note further that a higher p_1 weakly reduces the number of contracts with captive customers. Thus, total profit can only jump upwards (similarly for changes in p_2). Payoff security requires that for every mixed strategy profile, each supplier has a strategy that virtually guarantees the payoff he receives at that strategy profile, even if the other deviate slightly from it. Note that reducing slightly the price at worst only marginally reduces the supplier's payoff against small perturbations of the rival's price. It follows that the mixed extension of every price game is payoff secure. Therefore there exists at least one (possibly mixed) equilibrium for any z_1, z_2. For Statements 1–4, suppose now that z_1, z_2 are sufficiently close to zero.

Statement 1: First, note that using $\left(p_1^{nc}, p_2^{nc}\right)$, as given by (6) and (7), one can easily verify that (i) $x_1 \geq x$ and $x_2 \leq x$ if $t \leq \bar{t}^I$, (ii) $x_1 < 1$ if $t \geq \underline{t}_1^I$ and (iii) $x_2 > 0$ if $t \geq \underline{t}_2^I$.

Second, we check whether the solution $\left(p_1^{nc}, p_2^{nc}\right)$, as given by (6) and (7), yields positive profits in region A^I. We find that $\Pi_1\left(z_1, z_2, p_1^{nc}, p_2^{nc}\right)$ is positive if and only if $p_1^{nc} > [2q_4x_1 + q_3]/[q_2q_3]$. The last inequality holds in turn if and only if $x_1 < [tq_3q_2]/[2q_4] + 1$, which is always satisfied for the solution $\left(p_1^{nc}, p_2^{nc}\right)$. By symmetry, the same arguments hold for Π_2.

19 This result is stated as Corollary 5.2 in Reny (1999). The result directly generalizes previous mixed-strategy equilibrium results obtained for games with discontinuous payoffs, including Dasgupta and Maskin (1986). The main idea of Reny is to approximate the discontinuous payoff functions, not the strategy sets, by a sequence of continuous payoff functions. An advantage of the condition of better-reply security is that is often quite simple to check.

20 Let M_i denote the set of probability measures on the Borel subsets of X_i and extend u_i to $M = \times_{i=1}^N M_i$ by defining $u_i(\xi) = \int_X u_i(x)\, d\xi$ for all $\xi \in M$. Then $\bar{G} = (M_i, u_i)_{i=1}^N$ denotes the mixed extension of G.

Third, we check whether supplier 1 could gain by deviating from p_1^{nc} to a higher price such that $x_1 < x$ (monopoly case). Investigating the monopoly profits reveals that, whenever $x_1 < 1$ and $x_2 > 0$ holds for a duopoly, a monopolist would have an incentive to charge a lower price than p_1^{nc}. Hence, deviations from p_1^{nc} to a higher price cannot profitable. The same argument holds for supplier 2.

Forth, we need to check whether supplier 1 could gain by deviating from p_1^{nc} to a lower price such that $x_1 = 1$ (market coverage with respect to captive customers). Under this deviation, the highest profit for supplier 1 is obtained by charging the price $p_1^{nc} - q_4/[q_3q_2]$, which is the best response to p_2^{nc} given $x_1 = 1$. Subtracting the associated profit from the profit obtainable in the candidate equilibrium $\left(p_1^{nc}, p_2^{nc}\right)$ shows that the difference is positive if and only if $t > \underline{t}_1^I$. The same argument holds for supplier 2.

Finally, note that for z_1 and z_2 sufficiently close to zero, it is never profitable for any supplier to switch to a lower price such that $x = 1$ or $x = 0$. The argument is analogous to that for the original Hotelling game with linear travel costs (see d'Aspremont et al. 1979). This completes the proof of Statement 1, since we have shown that in the relevant parameter region no supplier has an incentive to deviate from the solution $\left(p_1^{nc}, p_2^{nc}\right)$.

Statement 2: First, note that using the solution $\left(p_1^c, p_2^c\right)$, as given by (8) and (9), one can easily verify that (i) $x_1 = 1$ if $t \leq \bar{t}_1^{II}$ and (ii) $x_2 = 0$ if $t \leq \bar{t}_2^{II}$.

Second, it is easy to show that $\Pi_1\left(z_1, z_2, p_1^c, p_2^c\right)$ and $\Pi_2\left(z_1, z_2, p_1^c, p_2^c\right)$ are non-negative if and only if $t \geq \underline{t}_1^{II}$ and $t \geq \underline{t}_2^{II}$.

Third, since the market is covered for captive customers in the candidate equilibrium $\left(p_1^c, p_2^c\right)$ as given by (8) and (9), there is no residual demand from non-captive customers for either supplier. Hence neither supplier could gain from charging a higher (local monopoly) price.

Forth, we need to check whether supplier 1 could gain by deviating from p_1^c to a higher price such that $x_1 < 1$ (no market coverage with respect to captive customers). Under this deviation, the highest profit for supplier 1 is obtained by charging $p_1^c + q_4/[q_3q_2]$, which is the best response to p_2^c given $x_1 < 1$. Subtracting the associated profit from the profit obtainable in the candidate equilibrium $\left(p_1^c, p_2^c\right)$ shows that the difference is negative if and only if $t < \bar{t}_1^{II}$. The same argument holds for supplier 2.

Finally, as argued in the proof of the previous statement, it is never profitable for any supplier to switch to a lower price such that $x = 1$ or $x = 0$ for all z_1 and z_2 not too large. Hence, $\left(p_1^c, p_2^c\right)$ as given by (8) and (9), is indeed an equilibrium of the price game in region A^{II}.

Statement 3: Brief inspection of (6) and (7) as well as (8) and (9) reveals that $\Pi_i\left(z_1, z_2, p_1^{nc}, p_2^{nc}\right) - \Pi_i\left(z_1, z_2, p_1^c, p_2^c\right) > 0$, $i = 1, 2$, in region $A^I \cap A^{II}$.

Statement 4: We will now check whether there exist equilibria which are asymmetric in the sense that only one supplier serves all of its captive customers. Suppose the solution of suppliers' maximization yields a pure-strategy equilibrium with $0 = x_2 < x < x_1 < 1$. Then equilibrium prices are

$$p_1^{as} = p_1^{nc} - \tfrac{2}{3q_3q_2}q_4, \quad p_2^{as} = p_2^{nc} - \tfrac{4}{3q_3q_2}q_4 \tag{16}$$

For (p_1^{as}, p_2^{as}) to be an equilibrium it is necessary that supplier 1 has no incentive to deviate by charging a lower price such that $x_1 = 1$ and that supplier 2 has no incentive to deviate by charging a higher price such that $x_2 > 0$. One can show that this is the case if and only if $\underline{t}_1^{as} \leq t \leq \overline{t}_2^{as}$, where

$$\underline{t}_1^{as} := \frac{1}{2}\frac{6Yq_3q_2-5q_4-6q_3}{q_3q_2(6-2z_1-z_2)}, \quad \overline{t}_2^{as} := \frac{1}{2}\frac{6Yq_3q_2-6q_3-7q_4}{q_3q_2(6-z_1-2z_2)} \tag{17}$$

However, it turns out that $\underline{t}_1^{as} > \overline{t}_2^{as}$ for z_1 and z_2 small enough, and that hence the parameter set where an asymmetric equilibrium might exist is empty. By symmetry, the same chain of arguments holds if we assume that there exists an equilibrium with $0 < x_2 < x < x_1 = 1$. It follows that there does not exist any other pure-strategy equilibrium than that of Type I or Type II in $A^I \cup A^{II}$. □

Appendix B

Analysis of location decisions

We will first construct a Pareto-perfect equilibrium with a high degree of differentiation and a high price level $\left(p_1^{nc}, p_2^{nc}\right)$. For this, it is convenient to consider a set of parameters in $\Omega^Y \times \Omega^t$ where, for $z_1 = z_2 = 0$ (i.e., maximal degree of differentiation) the game possesses both types of price equilibria: the high-price equilibrium of Type I and the low-price equilibrium of Type II. Such a set of parameters is the neighborhood $N(t^\circ, Y^\circ, \varepsilon)$:

$$N(t^\circ, Y^\circ, \varepsilon) : = \left\{(t, Y) : (t, Y) \in A_0^I, \left|t - t^\circ\right| < \varepsilon, \left|Y - Y^\circ\right| < \varepsilon\right\},$$
$$t^\circ : = \frac{11q_4}{2q_2q_3}, \quad Y^\circ := \frac{2q_3 + 25q_4}{2q_2q_3}, \quad \varepsilon > 0,$$

where $A_0^I := A^I$ for $z_1 = z_2 = 0$ and (t°, Y°) is an arbitrarily chosen point in $A_0^{II} := A^{II}$ and on the lower boundary of A_0^I.[21]

We will make use of the following definition:

Definition 4 Let $\left(t', Y'\right) \in N(t^\circ, Y^\circ, \varepsilon)$. If there exists a pair (z_1, z_2) such that for $z_1 = z_2 = z$, we have $\left(t', Y'\right) \in A^I \cap A^{II}$ and for ($z_1 \geq z$ and $z_2 > z$) or ($z_1 > z$ and $z_2 \geq z$) we have $\left(t', Y'\right) \in A^{II}$ and $\left(t', Y'\right) \notin A^I$, then z is defined to be $\hat{z}$.

That is, for each $\left(t', Y'\right) \in N(t^\circ, Y^\circ, \varepsilon)$, points $\hat{z}$ and $1 - \hat{z}$ are those locations of supplier 1 and 2, respectively, at which the game has both types of equilibria (Types I and II), but only Type II equilibrium if one of the suppliers, say supplier 1, moves slightly towards the center (as illustrated in Fig. 4 where supplier 2's location is $1 - \hat{z}$).

The idea is to show that $z_1 = z_2 = \hat{z}$ is part of a Pareto-perfect equilibrium for each $(t, Y) \in N(t^\circ, Y^\circ, \varepsilon)$. Throughout the analysis, we define, without loss of generality,

[21] We could have considered some other point in A_0^{II} and on the lower boundary of A_0^I. However, calculations were especially convenient for t° and Y° as used here.

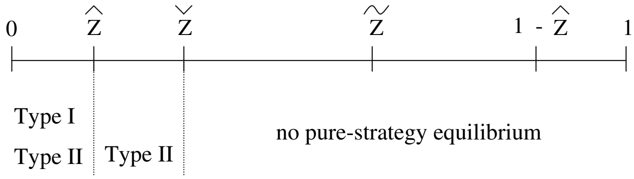

Fig. 4 Price equilibria for $z_2 = \hat{z}$

supplier 1 to be the supplier that unilaterally deviates from location $\hat{z}$, against supplier 2's location at $1 - \hat{z}$. Let $\check{z}$ be the largest position of supplier 1 such that there exists a pure-strategy price equilibrium (see Fig. 3).

The first step is to show that there exists a unique $\hat{z}$ for each $\left(t', Y'\right) \in N(t^\circ, Y^\circ, \varepsilon)$.

Lemma 1 *For each* $\left(t', Y'\right) \in N(t^\circ, Y^\circ, \varepsilon)$, *there exist a unique* $\hat{z}$, *as defined above.*

Proof of Lemma 1 Consider first point (t°, Y°). Note that (t°, Y°) is a point in the interior of A_0^{II}, where $A_0^{II} := A^{II}$ for $z_1 = z_2 = 0$, and belongs to the boundary $\underline{t}_1^I = \underline{t}_2^I$ of A_0^I if $z_1 = z_2 = 0$ (see Fig. 3). Observe firstly that $\max\left(\underline{t}_1^I, \underline{t}_2^I\right)$ is monotonically increasing in z_1 and monotonically increasing in z_2, and secondly that $\max(\underline{t}_1^{II}, \underline{t}_2^{II})$ is continuous in z_1 and continuous in z_2. Hence, if z_1 is slightly increased from $z_1 = z_2 = 0$, point (t°, Y°) ceases to be an element of A_0^I, but continues to be an element of A_0^{II}. Thus, for (t°, Y°), we have $\hat{z} = 0$.

By the same arguments and additionally by continuity of $\max\left(\underline{t}_1^I, \underline{t}_2^I\right)$ in z_1 and in z_2, existence and uniqueness of some $\hat{z}$ is ensured for each $\left(t', Y'\right) \in N(t^\circ, Y^\circ, \varepsilon)$. Recall that applicability of Proposition 1 (existence of Types I and II equilibria) is ensured for $z_1 = z_2 = \hat{z}$ small enough, which in turn is ensured in the neighborhood $N(t^\circ, Y^\circ, \varepsilon)$.

Lemma 2 *Suppose* $(t, Y) \in N(t^\circ, Y^\circ, \varepsilon)$ *and* $z_2 = \hat{z}$, *where* $\hat{z}$ *is small and defined as above. Then for each* z_1 *large enough such that no pure-strategy price equilibrium exists, the associated mixed-strategy equilibrium profit for supplier* 1 *is smaller than the profit obtainable in the Type-I equilibrium with* $z_1 = z_2 = \hat{z}$.

Proof of Lemma 2 The proof of this lemma is long and involved. However, since our approach to the analysis of games that do not possess a pure-strategy equilibrium in every subgame seems to be new, we will present it in some detail. To ease the exposition, we consider the parameters (t°, Y°) and hence $z_2 = \hat{z} = 0$. By continuity, the lemma holds for $z_2 = \hat{z} > 0$ and $\hat{z}$ sufficiently small, which is ensured in a neighborhood of (t°, Y°).

(I) Notation We will make use of the following notation. Let

$$h_1^c(p_1, p_2; z_1) := -q_4 + q_3 (p_1 q_2 - 1)\left(\frac{1}{2}(1 + z_1) + \frac{1}{2t^\circ}(p_2 - p_1)\right)$$

$$h_2^c(p_1, p_2; z_1) \; : \; = -q_4 + q_3 (p_2 q_2 - 1)\left(\frac{1}{2}(1 - z_1) - \frac{1}{2t^\circ}(p_2 - p_1)\right)$$

$$R_1^c(p_2) := \arg\max_{p_1} h_1^c(p_1, p_2; z_1) = \frac{11q_4 + 11z_1q_4 + 2q_2p_2q_3 + 2q_3}{4q_3q_2}$$

$$
\begin{aligned}
R_2^c(p_1) &:= \arg\max_{p_2} h_2^c(p_1, p_2; z_1) = \frac{11q_4 - 11z_1q_4 + 2q_2p_1q_3 + 2q_3}{4q_3q_2} \\
R_2^u(p_1) &:= p_1 - t^\circ(1 - z_1) \\
R_1^{nu}(p_2) &:= p_2 + t^\circ(1 - z_1) \\
h_2^u(p_2) &:= -q_4 + q_3(p_2q_2 - 1) \\
h_1^{nu}(p_1) &:= -q_4 + q_3(p_1q_2 - 1) z_1
\end{aligned}
$$

$\check{z}$ is defined to be the largest position of supplier 1 such that, given supplier 2 is located at $1 - \hat{z}$, a pure-strategy price equilibrium exists. From Lemma 1 we know that for $\hat{z} < z_1 \leq \check{z}$ and $z_2 = \hat{z}$ there exists only one pure-strategy price equilibrium, namely that of Type II. In such an equilibrium suppliers 1 and 2 earn $h_1^c\left(p_1^c, p_2^c; z_1\right)$, $h_2^c\left(p_1^c, p_2^c; z_1\right)$, respectively. Such an equilibrium ceases to exist if z_1 is large enough such that supplier 2 can gain from "undercutting", i.e., $h_2^c\left(p_1^c, p_2^c; z_1\right) < h_2^u(R_2^u(p_1^c))$. Hence $\check{z}$ solves the equation $h_2^c\left(p_1^c, p_2^c; \check{z}\right) = h_2^u(R_2^u(p_1^c))$. Note that $\check{z}$ is unique.

We define $p_1^a(z_1)$ as the solution to of the equation $h_2^c\left(p_1, R_2^c(p_1); z_1\right) = h_2^u\left(R_2^u(p_1)\right)$ with respect to p_1 for any given z_1. Furthermore, we define $\tilde{z}_1$ to be the solution of the equation $h_2^u\left(R_2^u(p_1^a(\tilde{z}_1))\right) = 0$. It is easy to check that $\check{z} < \tilde{z}_1$.[22] Now let (F_1, F_2) be a mixed-strategy equilibrium for a pair of positions $(z_1, 0)$ with $z_1 > \check{z}$. Let a_i and b_i be the respective smallest and largest prices in the support of F_i, and let $\pi_i^* = \Pi_i(F_1, F_2)$ be the respective expected equilibrium profit of supplier i with $i = 1, 2$. Finally, let $\Pi^* = \Pi_1\left(\hat{z}, \hat{z}, p_1^{nc}, p_2^{nc}\right) = \Pi_2\left(\hat{z}, \hat{z}, p_1^{nc}, p_2^{nc}\right)$ denote the equilibrium profit of both suppliers in the Type-I equilibrium, given that $z_1 = z_2 = \hat{z}$.

(II) Statements 1–3 We will prove the following three statements for any $z_1 > \check{z}$:

1. If $\check{z} < z_1 < \tilde{z}_1$, then $b_2 < Y$.
2. If $\tilde{z}_1 \leq z_1$ and $b_2 \geq Y$, then $\pi_1^* < \Pi^*$.
3. If $b_2 < Y$, then $\pi_1^* < \Pi^*$.

It is straightforward to see that $\pi_1^* < \Pi^*$ follows from these three statements.

(III) Proof of Statement 1 We will prove Statement 1 in three steps. In the first step, we derive a lower bound for a_1, namely, we show that $a_1 \geq R_1^c\left(R_2^u\left(p_1^a\right)\right)$. In the second step we calculate a_1 for the case of $b_2 \geq Y$. In the third step, we show that if $b_2 \geq Y$ then $a_1 < R_1^c\left(R_2^u\left(p_1^a\right)\right)$ for $z_1 < \tilde{z}_1$. This proves Statement 1 by contradiction.

Step 1 We first show that $R_1^c\left(R_2^u\left(p_1^a\right)\right)$ is a lower bound for a_1 by iterated elimination of strictly dominated strategies. We use four iterations: (i)–(iv).

[22] For the parameters (t°, Y°) and hence $\hat{z} = 0$, one obtains

$$
\begin{aligned}
\check{z} &:= 15 - 6\sqrt{6} \simeq 0.30306 \\
p_1^a &:= \frac{1}{2q_3q_2}\left(33q_4 + 2q_3 + 11z_1q_4 - 44\sqrt{z_1}q_4\right) \\
\tilde{z}_1 &:= \frac{12}{11} - \frac{2}{11}\sqrt{11} \simeq 0.48789
\end{aligned}
$$

(i) Clearly $a_2 \geq a_2' := \frac{q_3+q_4}{q_2 q_3}$ must hold, where a_2' is the solution to $h_2^u(a_2') = 0$, since for all $p_2^{nc} < a_2'$ supplier 2 earns negative profits for sure, which is dominated by $p_2^{nc} = Y$ with zero profits.

(ii) Now consider a_1, the smallest price in the support of supplier 1. Since z_2 is zero, supplier 1 cannot undercut supplier 2. Hence all $p_1^a < a_1' := \min\left(R_1^c(a_2'), R_1^{nu}(a_2')\right)$ are dominated by a_1' and consequently are not in the support of F_1.

(iii) Next we show that $a_2 \geq R_2^u\left(p_1^a\right)$. That is, we show that all $p_2^{nc} \leq R_2^u\left(p_1^a\right) - \varepsilon$, where ε is an arbitrarily small positive constant, are dominated by $R_2^c\left(p_1^a\right)$ for all p_1 in the support of F_1. We need to show that

$$\Pi_2\left(p_1, \underset{p_2 \leq R_2^u(p_1^a)-\varepsilon}{\operatorname{argmax}} \Pi_2(p_1, p_2)\right) < \Pi_2\left(p_1, R_2^c\left(p_1^a\right)\right)$$

First, we consider the case where $p_1 \geq p_1^a$. Note that $h_2^c\left(p_1, R_2^c\left(p_1^a\right); z_1\right) = h_2^u\left(R_2^u(p_1^a)\right)$ for $p_1 = p_1^a$ by the definition of p_1^a. Furthermore it is easy to check that: (1) $R_2^u\left(p_1^a\right) = \arg\max_{p_2 \leq R_2^u(p_1^a)} \Pi_2$ for $p_1 \geq p_1^a$; (2) $dh_2^u(R_2^u(p_1^a))/dp_1^a = 0$; (3) $\partial h_2^c/\partial p_1 > 0$; (4) $R_2^c\left(p_1^a\right) > R_2^u\left(p_1^a\right)$ for $p_1 \geq p_1^a$. It follows that

$$h_2^c\left(p_1, R_2^c\left(p_1^a\right); z_1\right) \geq h_2^u\left(R_2^u(p_1^a)\right) > \Pi_2\left(p_1, \underset{p_2 \leq R_2^u(p_1^a)-\varepsilon}{\operatorname{argmax}} \Pi_2(p_1, p_2)\right)$$
$$\text{for } p_1 \geq p_1^a$$

Where the last strict inequality follows from $dh_2^u(R_2^u(p_1))/dp_1^a > 0$.

Now we consider the case $p_1 \in [a_1', p_1^a[$. First, suppose $\arg\max_{p_2 \leq R_2^u(p_1^a)} \Pi_2(p_1, p_2) = R_2^u(p_1)$. Since $h_2^c\left(p_1^a, R_2^c\left(p_1^a\right); z_1\right) = h_2^u\left(R_2^u(p_1^a)\right)$ by the definition of p_1^a the stated domination argument follows from

$$\frac{dh_2^u\left(R_2^u(p_1)\right)}{dp_1} = q_2 q_3 > \frac{\partial h_2^c\left(p_1, R_2^c\left(p_1^a\right); z_1\right)}{\partial p_1} = q_2 q_3\left(1 - \sqrt{z_1}\right)$$

However, for p_1 well below p_1^a we have that $\arg\max_{p_2 \leq R_2^u(p_1^a)} \Pi_2\left(p_1', p_2\right) = R_2^u(p_1^a)$, with $\Pi_2\left(p_1', R_2^u(p_1')\right) = h_2^u\left(R_2^u(p_1')\right) = \Pi_2\left(p_1', R_2^u\left(p_1^a\right)\right) = h_2^c\left(p_1', R_2^u\left(p_1^a\right)\right)$ for some p_1' (note that $h_2^u\left(R_2^u(p_1)\right) \gtreqless h_2^c\left(p_1, R_2^u\left(p_1^a\right)\right)$ for $p_1 \gtreqless p_1'$).

Furthermore, for even smaller p_1' we have $h_2^c(p_1'', R_2^u\left(p_1^a\right)) = h_2^c(p_1'', R_2^c\left(p_1^a\right))$ for $p_1'' = [33q_4 + 2q_3 + 33z_1q_4 - 66\sqrt{z_1}q_4]/[2q_2q_3]$ (note that $h_2^c(p_1'', R_2^u\left(p_1^a\right)) \gtreqless h_2^c(p_1^a, R_2^c\left(p_1^a\right))$ for $p_1'' \gtreqless p_1$). It is a somewhat tedious but straightforward calculation to check that $p_1'' < a_1'$ for all $z_1 \in [\check{z}, \tilde{z}]$. This finally ensures that all $p_2 < R_2^u\left(p_1^a\right)$ are dominated by $R_2^c\left(p_1^a\right)$ as stated for (iii) in this step.

(iv) Now we want to show that $a_1 \geq R_1^c\left(R_2^u\left(p_1^a\right)\right)$. We do this by showing that all $p_1 < R_1^c\left(R_2^u\left(p_1^a\right)\right)$ are dominated by $R_1^c\left(R_2^u\left(p_1^a\right)\right)$ for all $p_2 \geq a_2 \geq a_2' = R_2^u\left(p_1^a\right)$, that is for all p_2 possibly in the support of F_2. Note that supplier 1 cannot profitably undercut supplier 2. Hence, $\Pi_1 = h_1^c$ and $\arg\max_{p_1 \le R_1^c(a_2')} \Pi_1(p_1, a_2') = R_1^c(a_2')$. Consequently, it is sufficient to verify for all $p_1 < R_1^c\left(a_2'\right)$ that

$$\frac{\partial h_1^c\left(p_1, p_2\right)}{\partial p_2} < \frac{\partial h_1^c\left(R_1^c\left(a_2'\right), p_2\right)}{\partial p_2}$$

The last inequality in fact holds for all $p_1 < R_1^c\left(a_2'\right)$ since $\partial^2 h_1^c\left(p_1, p_2\right)/\partial p_2 \partial p_1 > 0$.

Step 2 Suppose now that $b_2 \ge Y$. We will derive the value of a_1 for this case. Note that $b_2 \ge Y$ implies that $\Pi_2\left(F_1, F_2\right) = \Pi_2\left(F_1, p_2\right) = 0$ for all p_2 in the support of F_2. Hence, for a_1 we have $\max\left(h_2^u\left(R_2^u\left(a_1\right)\right), h_2^c\left(a_1, R_2^c\left(a_1\right)\right)\right) = 0$, for otherwise, supplier 2 could undercut supplier 1 and make a positive profit. Furthermore, it is easy to check that $h_2^c\left(p_1^a, R_2^c\left(p_1^a\right)\right) = h_2^u\left(R_2^u\left(p_1^a\right)\right) = 0$ must hold for $z_1 = \tilde{z}_1$. For $z_1 < \tilde{z}_1$ it is true that $h_2^c\left(p_1^a, R_2^c\left(p_1^a\right)\right) = h_2^u\left(R_2^u\left(p_1^a\right)\right) > 0$. Hence it follows that $a_1 < p_1^a$ for $z_1 < \tilde{z}_1$. Recall that $h_2^c\left(p_1^a, R_2^c\left(p_1^a\right)\right) = h_2^u\left(R_2^u\left(p_1^a\right)\right)$ holds by definition of p_1^a. Note that

$$0 < \frac{\partial h_2^c\left(p_1, R_2^c\left(p_1^a\right)\right)}{\partial p_1} < \frac{\partial h_2^u\left(R_2^u\left(p_1^a\right)\right)}{\partial p_1}$$

Hence, $h_2^c\left(a_1, R_2^c\left(a_1\right)\right) = 0 > h_2^u\left(R_2^u\left(a_1\right)\right)$, and the equation $h_2^c\left(a_1, R_2^c\left(a_1\right)\right) = 0$ determines a_1 for the case of $z_1 < \tilde{z}_1$ and $b_2 \ge Y$. For the parameters (t°, Y°) and hence $z_2 = \hat{z} = 0$ we obtain

$$a_1 = \frac{2q_3 + (4\sqrt{11} - 11\left(1 - z_1\right))q_4}{2q_2q_3} \tag{18}$$

for $z_1 < \tilde{z}_1$.

Step 3 Next, we show that if $b_2 \ge Y$ then $a_1 < R_1^c\left(R_2^u\left(p_1^a\right)\right)$ and thereby obtain a contradiction to the claim in Step 1. Using (18), straightforward calculations for the parameters (t°, Y°) and hence $z_2 = \hat{z} = 0$ yield

$$R_1^c\left(R_2^u\left(p_1^a\right)\right) - a_1 = \frac{1}{4}q_4\frac{55 - 8\sqrt{11} + 11z_1 - 44\sqrt{z_1}}{q_3q_2} \tag{19}$$

It is easy to verify that the right-hand side of (19) is strictly positive for all $z_1 < \tilde{z}_1$. The contradiction establishes the claim that $b_2 < Y$ if $z < \tilde{z}_1$.

(IV) Proof of Statement 2 Suppose that $z_1 \ge \tilde{z}_1$ and $b_2 \ge Y$. Note that in this case a_1 is the solution of $h_2^u\left(R_2^u\left(a_1\right)\right) = 0$. For parameters (t°, Y°) and hence $\hat{z} = 0$, we obtain $a_1 = [13q_4 - 11z_1q_4 + 2q_3]/[2q_3q_2]$. Since $da_1/dz_1 < 0$ we get $\pi_1^* < \Pi_1(a_1, Y)$ for all $z_1 \ge \tilde{z}_1$. Straightforward calculations for $z_1 = \tilde{z}_1$ yield $\Pi^* - \Pi_1(a_1, Y) = (147/44 - \sqrt{11})q_4 \simeq 0.0243q_4$. Hence, $\Pi^* > \pi_1^*$, which completes the proof of Statement 2.

(V) Proof of Statement 3 Suppose that $b_2 < Y$. Let p_1^{cc} be defined by $p_1^{cc} = R_2^c(p_1^{cc}) + t\left(1 - z_1 - \hat{z}\right)$, that is p_1^{cc} just prevents undercutting if supplier 2 plays $R_2^c(p_1^{cc})$. Then we have $b_1 \leq \gamma_1 := \min\left(p_1^c, p_1^{cc}\right)$, and $b_2 \leq \gamma_2 := R_2(\gamma_1)$. This follows from domination arguments, exactly as in Osborne and Pitchik (1987, Appendix 1(i)).

We will show by way of contradiction that $F_1\left(\hat{a}_1\right) > 0$, where $\hat{a}_1$ is defined as the solution of $h_2^u\left(\hat{a}_1, R_2^u\left(\hat{a}_1\right)\right) - h_2^c\left(\gamma_1, \gamma_2\right) = 0$.[23] Suppose that $F_1\left(\hat{a}_1\right) = 0$. We obtain the following chain of inequalities:

$$\int h_2^u\left(p_1, R_2^u\left(\hat{a}_1\right)\right) dF_1 = h_2^u\left(\hat{a}_1, R_2^u\left(\hat{a}_1\right)\right) \tag{20}$$
$$= h_2^c\left(\gamma_1, \gamma_2\right) \tag{21}$$
$$\geq h_2^c\left(\gamma_1, b_2\right) \tag{22}$$
$$\geq h_2^c\left(b_1, b_2\right) \tag{23}$$
$$> \int h_2^c\left(p_1, b_2\right) dF_1 \tag{24}$$
$$= \pi_2^*\left(F_1, b_2\right) = \pi_2^*\left(F_1, F_2\right) \tag{25}$$

where (20) is due to $F_1\left(\hat{a}_1\right) = 0$, and supplier 2 completely undercuts supplier 1 even at $p_1 = \hat{a}_1$, (21) follows from the definition of $\hat{a}_1$, (22) follows from the fact that γ_2 is best response to γ_1, and (23) follows from $b_1 \leq \gamma_1$, and (24) follows from $\partial h_2^c\left(p_1, b_2\right)/\partial p_1 > 0$. Finally, (25) follows from the fact that if supplier 2 plays b_2 then complete undercutting of supplier 1 does not occur even if supplier 1 plays b_1, since otherwise supplier 1 would never play b_1 with positive probability. This establishes the contradiction.

To complete the proof of Statement 3, assume that $\hat{a}_1$ is in the support of F_1 (i.e. $\hat{a}_1$ is not element of the set of F_1 -measure zero). Then we have

$$\pi_1^*\left(F_1, F_2\right) = \pi_1^*\left(\hat{a}_1, F_2\right) < h_1^c\left(\hat{a}_1, b_2\right) \leq h_1^c\left(\hat{a}_1, \gamma_2\right),$$

with

$$h_1^c\left(z_1, \hat{z}, \hat{a}_1, \gamma_2\right) = \begin{cases} h_1^c\left(z_1, \hat{z}, \hat{a}_1, p_2^c\right) & \text{for} \quad \check{z} \leq z_1 \leq \frac{3}{5} \\ h_1^c\left(z_1, \hat{z}, \hat{a}_1, R_2^c(p_1^{cc})\right) & \text{for} \quad \frac{3}{5} \leq z_1 \leq 1 \end{cases}$$

Note further that for $z_1 = 3/5$ we have $h_1^c\left(z_1, \hat{z}, \hat{a}_1, p_2^c\right) = h_1^c\left(z_1, \hat{z}, \hat{a}_1, R_2^c(p_1^{cc})\right)$ and $dh_1^c\left(z_1, \hat{z}, \hat{a}_1, R_2^c(p_1^{cc})\right)/dz_1 < 0$. By straightforward calculations we obtain for the parameters $\left(t^{\circ}, Y^{\circ}\right)$ that $h_1^c\left(z_1, \hat{z}, \hat{a}_1, p_2^c\right) < \frac{125}{44}q_4 = \Pi^*$ holds for all $\check{z} \leq z_1 \leq \frac{3}{5}$. If on the other hand $\hat{a}_1$ is element of the set of F_1-measure zero, then there exist some $a_1' < \hat{a}_1$ which is in the support of F_1, and the argument used before holds as well, since $h_1^c\left(a_1', \gamma_2\right) < h_1^c\left(\hat{a}_1, \gamma_2\right)$. This completes the proof of Statement 3. □

[23] For (t°, Y°) and hence $\hat{z} = 0$ we obtain $\hat{a}_1 = \frac{1}{36}\frac{297q_4+36q_3-264z_1q_4+11z_1^2q_4}{q_3q_2}$.

Proof of Proposition 2 We will show that for any $(t, Y) \in N(t^\circ, Y^\circ, \varepsilon)$ supplier 1 has no incentive to deviate from the following strategy, given supplier 2 also employs this strategy: *supplier i (with* $i, j = 1, 2, i \neq j$*) chooses* $z_i = \hat{z}$*, and*

1. if supplier j has chosen some z_j such that there exists a pure-strategy price equilibrium of Type I, then supplier i chooses p_i^{nc};
2. if supplier j has chosen some z_j such that there does not exist a pure-strategy price equilibrium of Type I, but a pure-strategy price equilibrium of Type II, then supplier i chooses p_i^c;
3. if supplier j has chosen some z_j such that there does not exist a pure-strategy price equilibrium, then supplier i chooses a mixed equilibrium strategy.

Note that in all price subgames the described strategies implement Nash equilibria. Hence, any deviation in the price subgames is not profitable. In the following we consider possible deviations in the location stage.

First, supplier 1 could choose to locate at $z_1 < \hat{z}$ in the case of $\hat{z} > 0$. The strategies described above yield a profit of $\Pi_1\left(\hat{z}, \hat{z}, p_1^{nc}, p_2^{nc}\right)$, while the deviation strategy yields $\Pi_1\left(z_1, \hat{z}, p_1^{nc}, p_2^{nc}\right)$. It is straightforward to check that $d\Pi_1\left(z_1, \hat{z}, p_1^{nc}, p_2^{nc}\right) /dz_1|_{dz_2=0} > 0$ in the neighborhood of (t°, Y°), which rules out any profitable deviations involving $z_1 < \hat{z}$. By Proposition 1 it has been established that $\left(p_1^{nc}, p_2^{nc}\right)$ is the Pareto-dominant price equilibrium for $z_1 < \hat{z}$.

Consider now possible deviations involving $z_1 > \hat{z}$. There are two cases to distinguish. First, deviations with $z_1 > \hat{z}$ such that there still exists a pure-strategy price equilibrium. Second, deviations to locations where there does not exist a pure-strategy equilibrium, that is, location where supplier 2 has an incentive to undercut supplier 1's price such that the whole market is served by supplier 2.

In the first case we know, by monotonicity of the boundary of A^I, that there does not exist a pure-strategy price equilibrium of Type I. Furthermore, apart from the equilibrium of Type II, there does not exist any other pure-strategy equilibrium. This follows from monotonicity of $\underline{t}_1^{as}$ and $\overline{t}_2^{as}$ in z_1 and z_2, where $\underline{t}_1^{as}$ and $\overline{t}_2^{as}$ are given in the proof of Proposition 1 (Statement 4). Note that these are the relevant boundaries where there might exist any other pure-strategy equilibria. Consequently, in the first case, the deviation yields at most $\Pi_1\left(z_1, \hat{z}, p_1^c, p_2^c\right)$ for supplier 1. We need to show that the difference $D := \Pi_1\left(\hat{z}, \hat{z}, p_1^{nc}, p_2^{nc}\right) - \Pi_1\left(z_1, \hat{z}, p_1^c, p_2^c\right)$ is positive for any $z_1 \in [0, 1]$. To do this, we analyze D for the parameters (t°, Y°) as a function of z_1 given $z_2 = \hat{z} = 0$. It is straightforward to check that D is quadratic in z_1 for any $z_1 \in [0, 1]$, and positive and decreasing in z_1 at the points $z_1 = 0$ and $z_1 = 1$ given $z_2 = 0$. Hence, given $z_2 = 0$, D must be positive for any $z_1 \in [0, 1]$. By continuity, this result holds for $z_2 = \hat{z} > 0$ and $\hat{z}$ sufficiently small, which is ensured in a neighborhood of (t°, Y°).

Next, consider locations where there exists no pure-strategy price equilibrium but only one in mixed strategies. By Lemma 2 it is ensured that no profitable deviation for supplier 1 to such locations exists.

Finally, note that $\hat{z}$ is unique. Hence, by the concept of Pareto-perfection, there is no other Pareto-perfect equilibrium. This finally completes the proof. □

Proof of Proposition 3 To prove the proposition, we will show that for any $(t, Y) \in N(t^\circ, Y^\circ, \varepsilon)$ supplier 1 has no incentive to deviate from the following strategy, given supplier 2 also employs this strategy: *supplier* i (with $i, j = 1, 2, i \neq j$) *chooses* $z_i = 0$ *and*

1. supplier j has chosen $z_j = 0$ then supplier i chooses p_i^{nc};
2. if supplier j has chosen some $z_j > 0$ such that there exists a pure-strategy price equilibrium of Type II, then supplier i chooses p_i^c;
3. if supplier j has chosen some z_j such that there does not exist a pure-strategy price equilibrium, then supplier i chooses some equilibrium distribution.

Note that, as in the previous proposition, the described strategies implement equilibria in all price subgames. Hence, any deviation in the price subgames is not profitable. Consider now deviations by supplier 1 to locations with $z_1 > 0$. As long as there exists an equilibrium of Type II in the price stage, the described strategies yield a profit of $\Pi_1 \left(z_1, \hat{z}, p_1^c, p_2^c\right)$ for supplier 1. In the proof if Proposition 2 we have shown for $\hat{z} = 0$ that this profit is lower than $\Pi_1 \left(0, 0, p_1^{nc}, p_2^{nc}\right)$, which is the profit obtainable with the described strategies. Moreover, for locations with $z_1 > 0$ where there exists no pure-strategy equilibrium, the same chain of arguments holds as in the proof of Proposition 2. □

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