# Second-mover Advantages in Dynamic Quality Competition

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**Abstract**

This paper explores a dynamic model of product innovation, extending the work of Dutta, Lach and Rustichini (1995). It is shown that if R&D costs for quality improvements are low, the dynamic competition is structured as a race for being the pioneer firm with payoff equalization in equilibrium, but switches to a waiting game with a second-mover advantage in equilibrium if R&D costs are high. Moreover, the second-mover advantage increases monotonically as R&D becomes more costly.



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# 1 Introduction

One of the widely held beliefs in business theory and practice is the advantage from being first in marketing a new product or technology. Venture capitalists, for instance, typically assess a pioneering firm as having a higher probability of survival as a follower firm.[1] However, research on games of timing has cast serious doubts on the reasonableness of this belief. As exemplified by Fudenberg and Tirole (1985), any first-mover advantage can be completely dissipated by the race to be first.[2] In their extension of Reinganum's (1981) duopoly model of technology adoption, they show that a first-mover advantage is not supported by subgame-perfect equilibrium strategies if firms are unable to precommit to future action. Recent theoretical work has consequently started to shift attention from first-mover to late-mover advantages: Dutta, Lach and Rustichini (1995) and Hoppe (2000a) demonstrate that a potential second-mover advantage can prevail as the subgame-perfect equilibrium outcome of duopolistic competition. Empirical evidence supports the results. Tellis and Golder (1996), for example, discover that the failure rate for pioneers is high. In contrast to the traditional view, their results suggest that following is, on average, better than pioneering. These findings raise the question: *when* do second-mover opportunities arise?

The present paper addresses this fundamental question. Examing a dynamic duopoly model of production innovation, as introduced by Dutta, Lach and Rustichini (1995), we find that the duration of technological competition and the costs of research and development (R&D) have countervailing effects on the existence and magnitude of a second-mover advantage in equilibrium.

[1]For a study on venture capitalists' decision making, see Shepherd (1999).

[2]In fact a similar argument has already been made by Karlin (1959, chapter 6), however without using the concept of subgame perfection.

Our result may be useful in providing a basis for further empirical work and, albeit more indirectly, for future public policy aimed at tuning the speed of innovation and diffusion.[3]

Dutta, Lach and Rustichini (1995) introduce a general duopoly model of product innovation in which the quality of a new product increases over time. At each point in time, each firm must choose whether to bring the currently available product to the market or whether to wait in order to market a product of higher quality. Assuming that a second-mover exists in equilibrium, the authors investigate how differences in firm characteristics may explain the firms' timing strategies and profitability. To give an illustrative example where a second-mover advantage may exist in equilibrium, Dutta, Lach and Rustichini also consider a more specific duopoly model of quality competition, based on that of Tirole (1988, p. 296). However, as we show in this paper, their model does not admit a second-mover advantage.

We show that if available quality increases costlessly over time, as in Dutta, Lach and Rustichini, the game is always a race to be the pioneer firm with payoff equalization in equilibrium. Nevertheless, by introducing positive R&D costs for improving quality and using a different approach as Dutta, Lach and Rustichini, we are able to demonstrate that a second-mover advantage emerges in equilibrium. That is, with two basic factors of product innovation, time (causing opportunity costs) and R&D effort (causing R&D expenditure), we find that if R&D cost are low and consequently the technological competition is mainly time-consuming, there is no second-

[3] As Stoneman and Diederen (1995) observe, technology policy is also frequently based upon the presumption that faster is always better. See their paper for an excellent survey on the literature on diffusion policy and actual policy initiatives. For recent studies of welfare issues and public policy with regard to adoption and diffusion of new technologies, see, e.g., Hoppe (2000a,b).

mover advantage. Conversely, if technological competition is mainly R&D effort-consuming, the game of quality competition changes its nature from a preemption to a waiting game with a second-mover advantage. Moreover, as the R&D cost parameter tends to infinity, the second-mover advantage converges to the high-quality advantage discovered by Aoki and Prusa (1997) and Lehmann-Grube (1997) in a static context. The analysis thereby links the results from the literature on static models of quality competition to the dynamic context.

There are several empirical studies which emphasize the importance of product quality improvements for the relative performance of firms.[4] Shankar, Carpenter, and Krishnamurthi (1998), for instance, analyze 13 brands in two pharmaceutical product categories and observe that second movers can overtake the pioneers through product innovation. Their results suggest that an innovative late entrant will enjoy a market potential, as measured by brand sales, at least as high as the pioneer's. Similarly, Berndt, Bui, Reiley, and Urban (1995) attribute a second-mover advantage in the U.S. antiulcer drug market, among other things, to better quality. Our finding that the duration of technological competition works against a possible second-mover advantage appears to be supported by a study of Lilien and Yoon (1990). Using a French data base of 112 new industrial products, they find evidence that the earlier a follower enters with a new product, i.e. the shorter the duration of technological competition, the better is the performance of that product.

In the next section we present the model. The impact of entry timing on the relative payoffs of the firms is discussed in Section 3. Section 4 considers

[4] For a comprehensive survey of the empirical literature on first-mover and second-mover advantages in the introduction of a new product, see Lieberman and Montgomery (1988, 1998) and Mueller (1997).

the case of costless R&D, while Section 5 deals with positive R&D costs. Section 6 concludes. All proofs are placed in the Appendix.

# 2 The model

On the supply side there are two firms, indexed by $i = 1, 2$, who can bring a new product to the market. The available product quality $s(t)$ is increasing over time $t$ by means of a deterministic and possibly costly research technology, where each firm's R&D costs per unit of time are $\lambda s$, with $\lambda \geq 0$; i.e. each firm invests continuously in R&D until it brings the product to the market. After a firm has entered the market, the quality of its product is fixed. We assume, as in Dutta, Lach and Rustichini (1995), that $s$ is proportional to $t$, and without further loss of generality that $t = s$. Variable costs of production are independent of quality and zero.

For the demand side, we use a model inspired by Tirole (1988). Each period each consumer buys at most one unit from either firm 1 or firm 2. Consumers differ in a taste parameter $\theta$, and they get in each period a net utility if they buy a quality $s_i$ at price $p_i$ of

$$U = s_i\theta - p_i \tag{1}$$

and zero otherwise.[5] A consumer of "taste" $\theta$ will buy if $U \geq 0$ for at least one of the offered price/quality combinations, and she will buy from the

[5] As a referee pointed out, for some goods the utility from not buying may possibly be negative. Such a case is neither considered in Tirole (1988) nor in Dutta, Lach and Rustichini (1995). Nevertheless, it should be interesting to check whether our results also hold when the value of the outside option is not zero. Our conjecture is that a negative valued outside option benefits the first mover due to higher monopoly profits. Conversely, a positive outside option may have the opposite effect. We leave this issue for future research.

firm that offers the best price/quality combination for her. Consumers are uniformly distributed over the range $[a, 1]$, where $1 > 2a \geq 0$.

To solve for the equilibrium of the price game for given qualities, two cases have to be distinguished with respect to the lower boundary of the consumer distribution, $a$:

**Case A.** $a$ is high enough such that the market is covered in equilibrium, as analyzed, for instance, by Tirole (1988) and Dutta, Lach and Rustichini (1995).

**Case B.** $a$ is sufficiently low such that some consumers do not buy in equilibrium, as analyzed, for instance, by Ronnen (1991) and Lehmann-Grube (1997).

Without loss of generality, we normalize currency units and the market size such that the equilibrium revenue flows per unit of time, defined for the interest rate $r = 1$, are

$$R_1^A = \left(\frac{1-2a}{3}\right)^2 (s_2 - s_1) \tag{2}$$

$$R_2^A = \left(\frac{2-a}{3}\right)^2 (s_2 - s_1) \tag{3}$$

in Case A, and

$$R_1^B = s_1 s_2 \frac{s_2 - s_1}{(4s_2 - s_1)^2} \tag{4}$$

$$R_2^B = 4s_2^2 \frac{s_2 - s_1}{(4s_2 - s_1)^2}. \tag{5}$$

in Case B. Notice that equilibrium revenues are independent of $a$ in Case B. In both cases the monopoly revenue per unit of time is

$$R_M = \frac{1}{4} s_1. \tag{6}$$

Each firm decides when to enter the market, given the best available quality to date and whether and when the rival has previously entered the market. The firm that enters first, i.e. the leader, is indexed by 1 and earns a flow of monopoly revenue of $R_M(s_1)$ from the time of its entry $s_1$ until $\hat{s}_2$, the optimal response by the second firm, i.e. the follower, who is indexed by 2. After $\hat{s}_2$ both firms earn a flow of duopoly Nash equilibrium revenues from price competition with vertically differentiated goods, $R_1^k(s_1, \hat{s}_2)$ and $R_2^k(s_1, \hat{s}_2)$, forever after, $k \in \{A, B\}$. Each firm observes its rival's entry instantaneously. To simplify matters, we follow Dutta, Lach and Rustichini in assuming that, if both firms attempt to enter first at any date, then only one firm - each with probability 1/2 - actually enters at that time and becomes the leader, while the other firm becomes the follower and may postpone its adoption. If the follower wants to choose joint entry, i.e. wants to enter consecutively but at the same instant, it may do so, and both firms get the same payoff.[6]

# 3 Payoff equalization or second-mover advantage

We use the subgame-perfect equilibrium in pure strategies as the solution for the game. Following Fudenberg and Tirole (1985), we take the optimal response of the follower, $\hat{s}_2(s_1)$, into account and write the leader's and the

[6] As an example, consider computer fairs that allow firms to announce the introduction of a new technology several times a year. If both firms plan to make an announcement at the same fair, one firm happens to have its press conference before the other with probability 1/2. The other firm observes the announcement of the first firm, and may decide to postpone its introduction date to a later fair. For problems of modelling closed-loop strategies in continuous-time games, see Simon and Stinchcombe (1989).

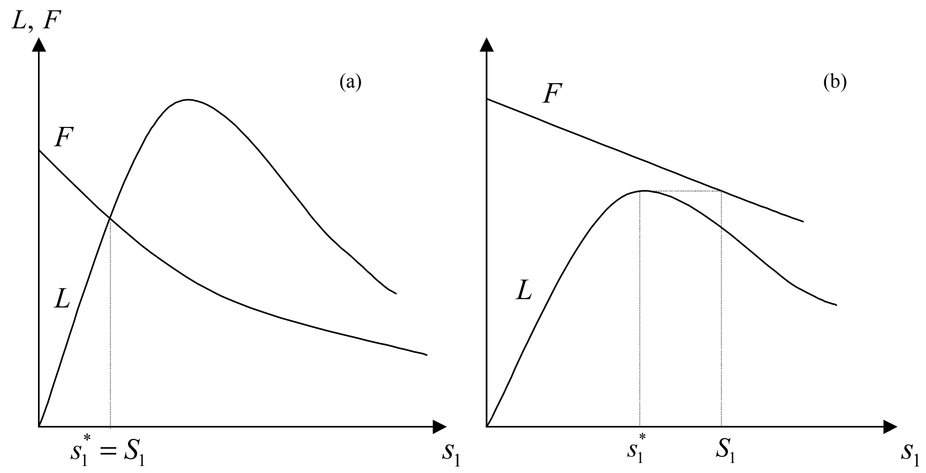


Figure 1:

follower's payoffs as functions of the leader's choice alone

$$L(s_1) = \int_{s_1}^{\hat{s}_2} e^{-\tau} R_M(s_1)\, d\tau + \int_{\hat{s}_2}^{\infty} e^{-\tau} R_1^k(s_1, \hat{s}_2) d\tau - \int_0^{s_1} e^{-\tau} \lambda \tau d\tau \quad (7)$$

$$F(s_1) = \int_{\hat{s}_2}^{\infty} e^{-\tau} R_2^k(s_1, \hat{s}_2) d\tau - \int_0^{\hat{s}_2} e^{-\tau} \lambda \tau d\tau \quad (8)$$

where $k \in \{A, B\}$ and the interest rate is $r = 1$.

Two potential payoff configurations are depicted in Figures 1a and 1b. Let $s_1^*$ and $S_1$ be defined by $F(S_1) = L(s_1^*)$ and $s_1^* \equiv \max\left\{\arg\max_{\tau \in [0, S_1]} L(\tau)\right\}$. That is, in Figure 1a, $s_1^*$ denotes the point in time where the $L$ curve and the $F$ curve intersect such that $s_1^* = S_1$. In Figure 1b, $s_1^*$ is arg max of the $L$ curve over the range $[0, S_1]$ with $L(s_1^*) < F(s_1^*)$ such that $s_1^* < S_1$.

Figure 1a and 1b illustrate two fundamentally different equilibrium outcomes. To see this, consider the situation depicted in Figure 1a and suppose for a while that the $L$ curve is single-peaked. The solution to the game is obtained by the following argument of Fudenberg and Tirole (1985). Each firm would like to be the first entrant at the arg max of the $L$ curve. Knowing this, firm $i$ has an incentive to adopt slightly earlier in order to preempt firm

$j$. But then firm $j$ could gain from preempting $i$. Backwards induction yields $s_1^* = S_1$ as the leader's equilibrium choice with equal payoffs for both firms in equilibrium, i.e. $L(s_1^*) = F(s_1^*)$. It is important to point out that the same equilibrium outcome is obtained when the $L$ curve is not single-peaked. In Hoppe and Lehmann-Grube (2001 [Theorem 1]) we prove that Fudenberg and Tirole's rent-equalization argument is applicable irrespective of the shape of the $L$ curve after $S_1$, and hence irrespective of the single-peakedness of the $L$ curve.[7] If the follower's best response function exists and the follower payoff is non-increasing in the leader's entry date, one needs to analyze the $L$ curve only up to the point $S_1$, with $s_1^* = S_1$ in the case of a preemption game.

Consider now the situation depicted in Figure 1b. Note that if both firms would wait longer than $S_1$ then one firm could gain from adopting at $s_1^*$. The leader's equilibrium choice is therefore $s_1^* < S_1$ which yields a second-mover advantage in the pure-strategy equilibrium, i.e. $L(s_1^*) < F(s_1^*)$.[8] This equilibrium is asymmetric. The competitors' expectations about the rival's strategies determine the equilibrium outcome. If, for example, firm $i$ believes that $j$ never enters first, $i$ may choose to be the first entrant. Likewise, if $j$ has the reputation of being likely to enter first, it may be optimal for $i$ to wait until $j$ has entered.[9]

The purpose of the paper is to investigate whether and when the game of dynamic quality competition admits a second-mover advantage in the pure-

[7] By contrast, the approach of Dutta, Lach and Rustichini (1995) requires single-peakedness of the $L$ curve, a condition which cannot be guaranteed analytically in Case B of the model.

[8] Games in which it is better to move second are often classified as war of attrition, see for instance Fudenberg and Tirole (1991). In the context of innovation, as in this paper, such games are generally denoted as waiting games or waiting contests.

[9] In the case where the game is structured as a waiting game, there is a continuum of mixed-strategy equilibria which are not considered here.

strategy equilibrium.

# 4 No second-mover advantage without R&D costs

In this section we consider the case of costless R&D, i.e. $\lambda = 0$. In Case A, in which the product market is covered in equilibrium, the $L$ curve can be shown to be single-peaked for costless R&D and the $F$-curve non-increasing in the leader's entry date $s_1$. Dutta, Lach and Rustichini (1995) claim that the competition in this case may be structured as a waiting game with a second-mover advantage in equilibrium, depending on the parameter $a$ that measures the minimal willingness to pay across customers. Proposition 1 shows that this is not correct: there is no second-mover advantage in equilibrium. Moreover, when R&D is costless, it can be shown that there is no second-mover advantage in Case B either.

**Proposition 1** *If R&D is costless, i.e.* $\lambda = 0$*, there is no subgame-perfect equilibrium with a second-mover advantage in Case A (market coverage) or in Case B (no market coverage).*

Proposition 1 indicates that both firms value the temporary monopoly position that is obtainable for the first entrant more than the strategic advantage of higher quality in the duopolistic stage. The dynamic quality competition is hence structured as a preemption game when R&D costs are zero. As a consequence, the payoffs of the early low-quality firm and the late high-quality firm are equated in the subgame-perfect equilibrium.

## 5 Second-mover advantage with R&D costs

In the following we consider only Case B, i.e. the case in which the market is not covered in equilibrium, and assume that the parameter $a$ is zero. We prefer this case for at least two reasons: (i) If firms choose their qualities before price competition takes place at the last stage of the game, as it is considered here, it is apparently impossible to restrict the parameter $a$ such that Case A is ensured to be the subgame-perfect equilibrium outcome of the game. On the other hand, if we restrict $a$ to being equal to zero, the equilibrium outcome is guaranteed to be Case B. (ii) The case where $a$ is equal to zero, and hence the market is not covered in equilibrium, corresponds to a standard linear and continuous demand function, while in Case A the implied demand function is discontinuous, which we regard as a rather artificial property.

In this section we show for the case in which the market is not covered, Case B, that a second-mover advantage will emerge if R&D activities are costly enough. More specifically, in the following proposition it is shown that if $\lambda$ becomes large the ratio $L/F$ eventually falls below 1.

**Proposition 2** *The payoff functions of the leader and the follower satisfy*

$$\lim_{\lambda\to\infty}\frac{L}{F}=\frac{\tilde{L}}{\tilde{F}} \tag{9}$$

*where*

$$\tilde{L} = s_1\hat{s}_2\frac{\hat{s}_2-s_1}{(4\hat{s}_2-s_1)^2}-\frac{\lambda}{2}s_1^2 \tag{10}$$

$$\tilde{F} = 4\hat{s}_2^2\frac{\hat{s}_2-s_1}{(4\hat{s}_2-s_1)^2}-\frac{\lambda}{2}\hat{s}_2^2 \tag{11}$$

*with* $\tilde{L}/\tilde{F}<1$ *in the subgame-perfect equilibrium.*

The intuition for this proposition is the following. When the R&D cost parameter, $\lambda$, gets large relative to the time-preference rate, $r$, technological competition becomes fast compared to the duration of product competition. That is, as $\lambda$ tends to infinitey, the firms select their qualities almost at the same point in time such that the leader and follower payoffs given by (7) and (8) converge to their limiting static form given by (10) and (11). The payoffs are in the limit the same as those used by Aoki and Prusa (1997) in a static game of quality competition with quadratic costs of quality. Aoki and Prusa show that the static model entails a high-quality advantage if firms choose their qualities simultaneously. By contrast, firms always choose qualities sequentially in our dynamic setting, where the low-quality firm is always the firm that chooses quality first. It turns out that the high-quality advantage persists in this case.

It follows from Proposition 2 that an increase in the R&D costs transforms the dynamic quality competition from a race to a waiting game with a high-quality/second-mover advantage in equilibrium. Such a transformation is ensured to take place if there exists a subgame-perfect equilibrium for any $\lambda$. This in turn is shown in Hoppe and Lehmann-Grube (2001). In the following we show, by applying a numerical algorithm, that this transformation is monotonic.[10]

**Result 1** *In the described game of dynamic quality selection there exists a unique value $\hat{\lambda} > 0$ such that for $\lambda \leq \hat{\lambda}$ the equilibrium outcome is payoff equalization, while for $\lambda > \hat{\lambda}$ the follower earns the higher payoff in equilib-*

[10] What is essential for numerical applications is that one only has to check for an equilibrium by examining the payoff curves $L$ and $F$ up to a finite point $S_1$. Furthermore, it has to be ensured that the equilibrium outcome is unique for any $\lambda$. We show in Hoppe and Lehmann-Grube (2001) that both preconditions are satisfied in our model.

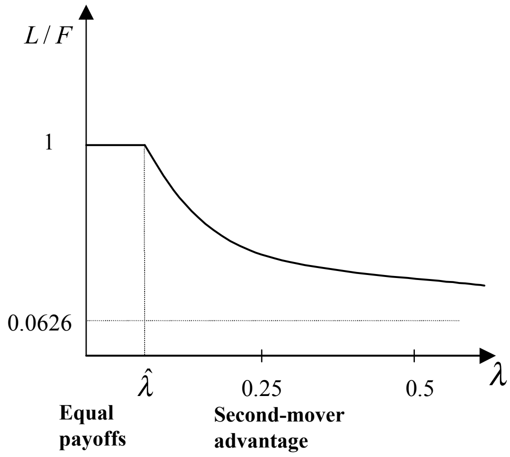


Figure 2:

*rium.*

The result is depicted in Figure 2, where we plot the ratio of equilibrium payoffs $L(s_1^*)/F(s_1^*)$ against a varying cost parameter $\lambda$.

Note that we use a technology for gradual innovation with two basic factors: time (causing opportunity costs) and R&D effort (causing R&D expenditure). If $\lambda$ is low, the main payoff-relevant factor is time. If on the other hand $\lambda$ is high, the main payoff-relevant factor is R&D effort. Our findings suggest that these factors have opposite impacts on the strategic nature of the technological competition. If R&D cost are low and consequently the technological competition is mainly time-consuming, then firms engage in a preemption game with no second-mover advantage. Conversely, if technological competition is mainly R&D effort-consuming, the dynamic game of quality selection changes its nature from a preemption to a waiting game with a high-quality/second-mover advantage.

The finding that R&D costs apparently work in favor of the high-quality follower may come as a surprise, since this firm invests in R&D for a longer period of time than the leader. But apart from this direct cost effect, an increase in the R&D costs per unit of time has the following indirect effects: (i) the high-quality follower innovates earlier with a negative impact on both the duration of the leader's monopoly period and the leader's duopoly revenues ($\partial R_1/\partial s_2 > 0$), and (ii) the first entry occurs earlier which has a positive impact on the duopoly revenues of the high-quality follower ($\partial R_2/\partial s_1 < 0$). Apparently, these indirect effects dominate the direct R&D cost effect when R&D costs are high.

# 6 Conclusion

Fudenberg and Tirole's (1985) work on games of preemption has important implications for the validity of the common held conception that it is better to be faster in the competition for innovation. Their result that any first-mover advantage is dissipated in equilibrium, coupled with substantial empirical evidence of late-mover advantages (Tellis and Golder (1996)), raises the question of *when* the technological competition is structured as a waiting game with a second-mover advantage or as a preemption game with payoff equalization.

We have extended a duopoly model of dynamic quality competition, as introduced by Dutta, Lach and Rustichini (1995), by incorporating R&D costs and found that the duration of the technological competition and the costs of R&D have countervailing effects on the relative performance of firms. A second-mover advantages occurs when R&D cost considerations are the dominant factor of the timing decisions of the firms. Conversely, if technological competition is primarily time-consuming, firms obtain equal payoffs in equilibrium.

One can show that the result that R&D costs are responsible for a second-mover advantage is robust to generalizations of the model to any convex R&D cost function in the limiting static case. It seems worth investigating other models of innovation and entry timing, with the objective to derive testable hypotheses regarding the impact of parameters determining the R&D costs and the duration of technological competition on the existence and magnitude of second-mover advantages.

# Appendix

**Proof of Proposition 1.** **Case A:** We will show that the existence of a second-mover advantage in the subgame-perfect equilibrium of Case A is not consistent with market coverage. The equilibrium revenues for Case A are given by $R_1^A$ and $R_2^A$. Substituting into (7) and (8), Dutta, Lach and Rustichini (1995) show that $F(s_1^*) > L(s_1^*)$, where $s_1^*$ is the global maximum of $L(s_1)$, only if

$$a < \xi \equiv 2 - \frac{3}{2}\sqrt{e(1 - \frac{1}{e})} \tag{12}$$

However, the following market-coverage condition has to be satisfied in equilibrium (see Tirole (1988), p. 296, Assumption 2):

$$\begin{aligned} \frac{1-2a}{3}(s_2 - s_1) &\leq as_1 \\ &\Leftrightarrow \\ \frac{s_2 - s_1}{s_1 + 2s_2} &\leq a. \end{aligned}$$

In the following it is shown that $\xi < (s_2 - s_1) / (s_1 + 2s_2)$ holds in equilibrium, which implies that there can be no second-mover advantage in Case A.

The optimal reaction of the follower is (see Dutta et al. (1995), p. 571)

$$\hat{s}_2 = 1 + s_1 \tag{13}$$

Solving the maximization problem of the leader yields

$$s_1^* = \frac{9 - 13e^{-1} + 16ae^{-1}(1-a)}{9(1-e^{-1})} \tag{14}$$

Using (13) and (14), it is not hard to check that $\xi < (\hat{s}_2 - s_1^*)/(s_1^* + 2\hat{s}_2)$ is equivalent to $0 < 3(1-e^{-1}) - \xi(15 - 19e^{-1} + 16ae^{-1}(1-a))$, which is true if $0 < 3(1-e^{-1}) - \xi(15 - 19e^{-1} + 4e^{-1}) \simeq 1.58$.

**Case B:** In this case we show that the $L$ curve and the $F$ curve intersect once in the range $[0, \check{s}_1]$ for some $\check{s}_1$ and that the $L$ curve has no local maximum over that range. The following conditions (i-iv) ensure that the timing competition is structured as a preemption game with rent equalization because we know from the analysis in Hoppe and Lehmann-Grube (2001) that $F$ is decreasing in the leader's choice, and that it is therefore not necessary to examine the $L$ curve and the $F$ curve for $s_1 \geq \check{s}_1$. (i) We need to find some $\check{s}_1$ such that $F(\check{s}_1) < L(\check{s}_1)$, (ii) we show that $L(0) < L(\check{s}_1)$ and $L(0) < F(0)$, (iii) we show that $L(s_1)$ is continuous over the range $[0, \check{s}_1]$, and finally (iv) we show that $L(s_1)$ is increasing over the range $[0, \check{s}_1]$.

As a preliminary step, we need to solve the follower's maximization problem. Let $\pi_2(s_1, s_2)$ denote the follower's payoff as a function of both innovation dates. The follower's first-order condition is:

$$\begin{aligned} &\frac{\partial \pi_2}{\partial s_2} = \frac{\partial}{\partial s_2}\left(\int_{s_2}^{\infty} e^{-\tau} R_2^B d\tau\right) = 0 \\ \Rightarrow\ & 4s_2^3 - 5s_2^2 s_1 - 4s_2^2 + s_2 s_1^2 + 3s_1 s_2 - 2s_1^2 = 0 \end{aligned} \tag{15}$$

Solving this equation with respect to $s_2$ yields

$$s_{2_1} = p - \frac{-q}{p} + \frac{1}{3} + \frac{5}{12}s_1$$

$$
\begin{aligned}
s_{2_{2,3}} &= -\frac{1}{2}p + \frac{1}{2}\frac{-q}{p} + \frac{1}{3} + \frac{5}{12}s_1 \pm \frac{1}{2}i\sqrt{3}\left(p + \frac{-q}{p}\right), \text{ where} \\
q &= \frac{1}{36}s_1 + \frac{13}{144}s_1^2 + \frac{1}{9} \\
p &= \sqrt[3]{p_1 + \frac{1}{288}\sqrt{p_2}} \\
p_1 &= \frac{1}{72}s_1 + \frac{65}{288}s_1^2 + \frac{35}{1728}s_1^3 + \frac{1}{27} \\
p_2 &= 504s_1^3 + 4029s_1^4 + 1104s_1^2 + 702s_1^5 - 27s_1^6
\end{aligned}
$$

It is obvious that both $p_1$ and $p_2$ are positive for $0 \leq s_1 \leq 1$ and therefore that $p$ is real in that range. It follows that the only real solution for $s_1 < 1$ is $\hat{s}_2 \equiv s_{2_1} = p + q/p + 1/3 + 5/12s_1$, which is the follower's best response, and a continuous function.

Now we proceed with steps (i)-(iv) as described above.

(i) Let $\check{s}_1 = 0.9$. Then $s_2(\check{s}_1) \simeq 1.7567$, and $L(\check{s}_1)/F(\check{s}_1) \simeq 1.0572$.

(ii) $0 = L(0) < L(\check{s}_1) \simeq 0.0514$, and furthermore, straightforward calculations reveal that $F(0)$ is positive and hence $L(0) < F(0)$.

(iii) It has been shown that $\hat{s}_2$ is a continuous function. This is sufficient for the continuity of $L(s_1)$.

(iv) It is left to verify that $L(s_1)$ has no interior local maximum before $\check{s}_1$. The derivative of $L$ is

$$
\begin{aligned}
L' &= \frac{e^{-s_1} - e^{-\hat{s}_2}}{4} - \frac{1}{4}s_1 e^{-s_1} + e_2^{-\hat{s}_2}\hat{s}_2^2 \frac{4\hat{s}_2 - 7s_1}{(4\hat{s}_2 - s_1)^3} \\
&\quad + \hat{s}_2' e^{-\hat{s}_2}\left(\frac{1}{4}s_1 + s_1^2 \frac{2\hat{s}_2 + s_1}{(4\hat{s}_2 - s_1)^3} - s_1\hat{s}_2 \frac{\hat{s}_2 - s_1}{(4\hat{s}_2 - s_1)^2}\right)
\end{aligned}
$$

which is continuous as long as $\hat{s}_2'$, the derivative of $\hat{s}_2(s_1)$, is continuous. We know from Hoppe and Lehmann-Grube (2001) that the follower's payoff, $\pi_2(s_1, s_2)$, is single-peaked in his own choice, which implies that $\partial^2\pi_2/\partial s_2^2 < 0$ holds at the maximum. $\hat{s}_2' = -\left(\partial^2\pi_2/\left(\partial s_2 \partial s_1\right)\right)/\left(\partial^2\pi_2/\partial s_2^2\right)$ is therefore continuous. We obtain $L'(\check{s}_1) = A$, where $A$ is strictly positive. Next, we check that $L'(z \cdot \varepsilon) > A$, where $z$ is an integer with $z \in \{0...n\}$, $n$ is large, and $\varepsilon = \check{s}_1/n$ is the size of the steps of calculation. This ensures, by the continuity of $L'$, that $L'(s_1) > 0$ for all $s_1 \in [0, \check{s}_1]$, as stated. ■

**Proof of Proposition 2.** $\lambda$ is defined as R&D costs per quality unit and per unit of time. An increase in $\lambda$ is equivalent to a shortening of the units in which time is measured while keeping $\lambda$ constant. This is in turn equivalent to a decrease in the interest rate $r$, while keeping $\lambda$ constant. In the following we derive the limit of the payoffs by keeping $\lambda$ constant, while $r$ approaches zero. The proof relies on appropriate rescaling of the units which are used in the model.

We have normalized the units to measure quality $s$ to be equal to the units in which time is measured, i.e. $t = s$. If $r$ becomes small, that is the units in which time is measured get small, for example months instead of years and so forth, quality units can be appropriately rescaled such that $t = s$ still holds for every choice of the units in which time is measured. Revenues are measured in currency units per unit of quality and per unit of time. We can adjust currency units appropriately to maintain the revenues $R_M$, $R_1$, and $R_2$ defined for the original units of time. It follows that revenues are proportional to the length in which units of time are measured and hence to the interest rate $r$. Thus,

$$\begin{aligned}
\tilde{L}(s_1) &\equiv \lim_{r \to 0} \left( \frac{e^{-rs_1} - e^{-r\hat{s}_2}}{r} R_M r + \frac{e^{-r\hat{s}_2}}{r} R_1^B r - \int_0^{s_1} e^{-r\tau} \lambda \tau d\tau \right) \\
&= s_1 \hat{s}_2 \frac{\hat{s}_2 - s_1}{(4\hat{s}_2 - s_1)^2} - \frac{\lambda}{2} s_1^2 \qquad (16) \\
\tilde{F}(s_1) &\equiv \lim_{r \to 0} \left( e^{-r\hat{s}_2} R_2^B r - \int_0^{\hat{s}_2} e^{-r\tau} \lambda \tau d\tau \right) \\
&= 4\hat{s}_2^2 \frac{\hat{s}_2 - s_1}{(4\hat{s}_2 - s_1)^2} - \frac{\lambda}{2} \hat{s}_2^2 \qquad (17)
\end{aligned}$$

where $\tilde{L}$ and $\tilde{F}$ are measured in infinitesimal currency units.

Solving the maximization problem of the leader, while taking into account the follower's optimal response, yields two equations in two variables:

$$\frac{\partial}{\partial s_2}\left(R_2^B - \frac{\lambda}{2}s_2^2\right) = 0 \Rightarrow s_1\lambda = \frac{16b^2 - 12b + 8}{(4b-1)^3} \tag{18}$$

$$\frac{d}{ds_1}\left(R_1^B - \frac{\lambda}{2}s_1^2\right) = \frac{\partial R_1^B}{\partial s_1} - \lambda s_1 + \frac{\partial R_1^B}{\partial s_2}\cdot\frac{d\hat{s}_2}{ds_1} = 0$$

$$\Rightarrow \frac{4b^3 - 7b^2}{(4b-1)^3} - \lambda s_1 + \frac{2b+1}{(4b-1)^3}\cdot\frac{16 + 3\lambda s_1 (4b-1)^2 - 12b}{12 + 12\lambda s_1 (4b-1)^2 - 32b} = 0 \tag{19}$$

where $s_1\lambda$ is one variable and $b \equiv \hat{s}_2/s_1$ the other, and $d\hat{s}_2/ds_1$ is obtained by differentiating the follower's first-order condition (18). The solution of (18) and (19) in these two variables is independent of $\lambda$. Substituting the first equation into the second yields:

$$64b^5 - 432b^4 + 644b^3 - 783b^2 + 394b - 166 = 0$$

It is easily shown that this last equation has only one solution for $b > 1$, namely:

$$\frac{\hat{s}_2}{s_1} = b \simeq 5.2347$$
$$\lambda s_1 \simeq 0.0484$$

Finally, the ratio of the payoffs depends only on the two variables $s_1\lambda$ and $\hat{s}_2 = bs_1$ and is hence in equilibrium independent of $\lambda$:

$$\frac{\tilde{L}}{\tilde{F}} = \frac{b\frac{b-1}{(4b-1)^2} - \frac{1}{2}s_1\lambda}{4b^2\frac{b-1}{(4b-1)^2} - \frac{1}{2}b^2 s_1\lambda} \simeq 0.0626 \tag{20}$$

■

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