

The Strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games*

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Abstract

We establish the strategic equivalence of a variety of rent-seeking contests, innovation tournaments, and patent-race games. The results allow us to disentangle negative and positive externalities, and to apply theorems and results intended for rent-seeking games to other games, and vice versa. We conclude with several examples that highlight the practical utility of our results. (**JEL Numbers: D00, L00, D72; Keywords: Contest, Rent Seeking, Innovation Tournament, R&D, Patents**).

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1 Introduction

Over the past two decades, a number of papers have independently analyzed games of rent-seeking and innovation.¹ While it is well-known (cf. Baye, Kovenock, and de Vries, 1996) that many of these games have similar “all-pay” structures in which winners and losers alike forfeit the resources expended to win the prize, a formal analysis of the relationship between rent-seeking and innovation games is lacking. This paper represents a first attempt to formalize the “duality” between rent-seeking contests and games of innovation.

The models of rent-seeking considered in this paper include the seminal model of Gordon Tullock, as well as more general contests such as those in Dixit (1987) and Skaperdas (1996). In addition, our analysis covers two different classes of innovation games: patent races, which are typically used to model the competition to be first, and innovation tournaments, which are most frequently used to formalize the competition to be best. The paper considers the classic patent race game as pioneered by Loury (1979) and Dasgupta and Stiglitz (1980), and contributes to the literature on innovation tournaments by offering a new tournament model in which R&D efforts determine not only the probability of winning, but also the value of the winner’s

¹For a discussion of the alternative approaches to the modeling of R&D, see e.g. Mortensen (1982), Reinganum (1989) or Taylor (1995); for a comprehensive survey of the rent-seeking literature, see Nitzan (1994).

prize.²

We introduce the endogenous-prize formulation for two reasons. First, we wish to cover the two extreme forms of innovation competition: patent races where R&D accelerates the innovation of a fixed value, and tournaments where R&D increases the *ex post* value of the innovation but where timing is no issue. Second, in many economic situations, R&D efforts affect the profits that are associated with having the best idea. Venture capitalists, for example, frequently run endogenous-prize tournaments by allowing only the best entrepreneur to go to the initial public offering (IPO) market.³ Other examples include research tournaments in which the firm with the best idea wins an exclusive right for commercializing its idea, such as the tournament recently sponsored by the Federal Communications Commission to develop the best technology for high-definition television (HDTV) where the technology of the winner was chosen as the HDTV standard.

Despite their obvious presence, endogenous-prize innovation tournaments have not been discussed extensively in the theoretical literature. We model the R&D competition to be best as a process where each firm runs parallel experiments (funded through R&D) and chooses to go forward with its most valuable project. This view of R&D dates back at least to Nelson (1961), who notes that parallel-path strategies are

²Much of the existing literature treats the value of the prize as an exogenous parameter; see Taylor (1995) and Fullerton and McAfee (1999) for studies of sponsored research tournaments.

³See, for example, Aoki (2001, Chapter 14).

a standard practice in many industrial laboratories. Our analysis of R&D extends the endogenous prize “parallel-path” monopoly model of Evenson and Kislev (1976) to an oligopolistic setting in which multiple firms engage in R&D. Our model is similar to the oligopoly model of Fullerton and McAfee (1999), except that we allow a firm’s R&D experiments to impact not only the probability of winning, but also the *ex post* value of the prize.

We identify conditions under which innovation games are strategically equivalent to contests and, in particular, to the Tullock game. The duality results allow us to disentangle different external effects of R&D activity. It is found that innovation tournaments exhibit not only negative externalities to R&D due to the incentive to win the tournament, but also positive externalities due to the incentive to be better than the rivals. That is, apart from the well-known “business-stealing effect,” there is a countervailing “leap-frogging effect” on the value of winning the prize. Furthermore, we find that this leap-frogging effect may be exactly offset by diseconomies of density in the total number of R&D experiments. In such a case, the innovation tournament reduces to the classical Tullock game, which in turn is shown to be strategically equivalent to a classical patent race game under certain conditions.

The duality results also permit one to apply results derived in the contest and rent-seeking literatures to the innovation context, and vice versa. For example, applying Chung’s (1996) results for rent-seeking contests to the innovation context allows us to conclude that the innovation tournaments considered in this paper always involve

excessive R&D investments: The negative business-stealing effect dominates the positive leap-frogging effect. We conclude the paper with a number of other examples that highlight some practical applications of our results.

2 Three Classes of Games

This section presents the contests and innovation games considered in this paper. The interested reader may consult the literature and surveys identified in the Introduction for a more extensive discussion of the various games.

2.1 Rent-Seeking Contests

Let x_1, x_2, \dots, x_n denote the effort levels of the n players, let $v(x_1, x_2, \dots, x_n)$ represent the value of the contested prize, and let $p_i(x_1, x_2, \dots, x_n)$ denote the probability that player i wins the contest. Player i 's expected payoff is

$$\pi_i(x_1, x_2, \dots, x_n) = v(x_1, x_2, \dots, x_n) p_i(x_1, x_2, \dots, x_n) - x_i.$$

In the sequel, we shall let $C(v, p, n)$ denote this class of contests.

Notice that Dixit (1987) corresponds to the special case where v is constant and p_i is a twice continuously differentiable, symmetric function of players' efforts. The all-pay-auction (cf. Baye, Kovenock and de Vries, 1996) corresponds to the special case where v is constant and p_i is discontinuous (the player with the greatest x_i wins with probability one). The most celebrated special case is, of course, the Tullock

rent-seeking game in which player i 's expected payoff is

$$\pi_i(x_1, x_2, \dots, x_n) = v \frac{x_i^R}{\sum_{j=1}^n x_j^R} - x_i,$$

where v is constant and $R > 0$. We denote the family of Tullock games as $T(v, R, n)$. Equilibria for $T(v, R, n)$ (including the case where $R > 2$) are known; see Baye, Kovenock and de Vries (1994).

2.2 Innovation Tournaments

Consider n firms who compete to enter a new market with an innovation. Suppose the values of ideas for the new product are distributed according to F on $[0, 1]$. The ideas are borne from innovation activities, which we model as experiments whose outcomes are random in that firms cannot pinpoint ideas that will be valuable. A firm can, however, increase its likelihood of having a valuable idea by increasing its R&D effort, which is represented by increasing the size of the sample it draws from F . The sample size for firm i is determined by the number of scientists that firm i employs, and is denoted by m_i . We assume that the firm discovering the best idea y wins a patent of value y , while all other firms gain nothing.⁴

The firms simultaneously choose how many ideas to draw from F . Each draw costs c , so that the total cost to firm i of m_i draws is cm_i . We assume that $c < \int_0^1 yf(y) dy$, where f denotes the probability density function of valuable ideas. Thus, R&D costs

⁴The existing literature on innovation tournaments assumes that the *ex post* value of the prize is exogenously given (Taylor, 1995; Fullerton and McAfee, 1999).

for a single draw are less than the expected value of the idea associated with it. This is a necessary condition for a firm to invest in one or more ideas.

Let $z_i \equiv \max \{y_{i1}, y_{i2}, \dots, y_{im_i}\}$ denote the best idea of firm i , let $\sum_j m_j$ denote the total number of scientists hired by all firms, and let $\sum_{j \neq i} m_j$ denote the number of scientists hired by all firms but i . The value to firm i of hiring m_i scientists when the rivals have hired $\sum_{j \neq i} m_j$ scientists is

$$\Pr \left(\max_{j \neq i} \{z_j\} \leq z_i \right) z_i,$$

which is a random variable. To find the expectation of this random variable, note that $\Pr(\max_{j \neq i} \{z_j\} \leq z_i) = (F(z_i))^{\sum_{j \neq i} m_j}$ and $\Pr(\max_k \{y_{ik}\} \leq z_i) = (F(z_i))^{m_i}$, $k = 1, 2, \dots, m_i$. Thus, the expected value to firm i of hiring m_i scientists when the rivals hire $\sum_{j \neq i} m_j$ scientists is

$$\begin{aligned} v(m_1, m_2, \dots, m_n) &= \int_0^1 \left(\Pr \left(\max_{j \neq i} \{z_j\} \leq z_i \right) z_i \right) d(F(z_i))^{m_i} \\ &= \int_0^1 (F(z_i))^{\sum_{j \neq i} m_j} z_i m_i (F(z_i))^{m_i-1} f(z_i) dz_i, \end{aligned} \quad (1)$$

and firm i 's expected payoff in the innovation tournament is given by

$$\pi_i(m_1, m_2, \dots, m_n) = v(m_1, m_2, \dots, m_n) - cm_i.$$

Let $I(F, c, n)$ denote this family of innovation games.

This family of innovation games has the property that the *ex post* value of the prize is endogenous and equals the value of the best (random) discovery. On the surface, this is in contrast to the Fullerton-McAfee (1999) research tournament model where

the players' discovery efforts affect the probability of winning, but not the value of the prize. As will be shown below, however, the family $I(F, c, n)$ of innovation games includes innovation tournaments that are strategically equivalent to the Fullerton-McAfee formulation. Thus, the class of innovation games we consider encompasses a variety of different R&D settings.

2.3 Patent Races

Consider the classic model of a patent race as pioneered by Loury (1979) and Dasgupta and Stiglitz (1980). There are n firms. The firm that innovates first will obtain a patent of infinitely long life, while all other firms gain nothing. Let v denote the value of the patent. It is assumed that if firm i chooses a lump-sum R&D investment x_i , its probability of making a discovery on or before time t is

$$1 - e^{-h(x_i)t}$$

where $h(x_i)$ is the "hazard rate" of firm i , that is, firm i 's conditional probability of making a discovery between time t and time $t + dt$, given that no innovation has occurred at or before t . The function h is defined on $[0, 1]$ and assumed to be increasing and concave.⁵ Letting r denote the interest rate, firm i 's payoff in the patent race is given by

$$\pi_i(x_1, x_2, \dots, x_n) = \int_0^\infty h(x_i) v e^{-(\sum_{j=1}^n h(x_j) + r)t} dt - x_i,$$

⁵The assumption of concavity is made to ensure uniqueness of the equilibrium.

Let $P(h, v, r, n)$ denote this family of patent races.

3 Results

We first establish a relationship between innovation tournaments $\gamma \in I(F, c, n)$ and contests $\Gamma \in C(v, p, n)$. Let $\Sigma_j x_j = \Sigma_{j=1}^n x_j$ denote the total effort of all players and $\Sigma_{j \neq i} x_j = \Sigma_{j \neq i}^n x_j$ denote the effort of all players except i .

Theorem 1 *An innovation game $\gamma \in I(F, c, n)$ is strategically equivalent to a contest $\Gamma \in C(v, p, n)$ with success function*

$$p_i(x_1, x_2, \dots, x_n) = \frac{x_i}{\Sigma_j x_j},$$

prize function

$$v(x_1, x_2, \dots, x_n) = \frac{1}{c} \left[1 - \int_0^1 (F(z))^{\Sigma_j x_j} dz \right],$$

and where each player's strategy space is $\{0, 1, 2, \dots\}$. Furthermore, this prize function is increasing in individual effort (x_i), rivals' effort ($\Sigma_{j \neq i} x_j$), and total effort ($\Sigma_j x_j$), but at a decreasing rate.

Proof. In an innovation game $\gamma \in I(F, c, n)$, firm i 's expected payoff can be written as

$$\pi_i(m_1, m_2, \dots, m_n) = m_i \int_0^1 (F(z))^{\Sigma_j m_j - 1} z dF(z) - cm_i.$$

Let $w = z$ and $u = (F(z))^{\Sigma_j m_j}$, and integrate by parts to obtain:

$$\begin{aligned}
\pi_i(m_1, m_2, \dots, m_n) &= \frac{m_i}{\sum_j m_j} \int_0^1 w du - cm_i \\
&= \frac{m_i}{\sum_j m_j} \left(uw|_0^1 - \int_0^1 u dw \right) - cm_i \\
&= \frac{m_i}{\sum_j m_j} \left[(F(z))^{\sum_j m_j} z|_0^1 - \int_0^1 (F(z))^{\sum_j m_j} dz \right] - cm_i \\
&= \frac{m_i}{\sum_j m_j} \left[1 - \int_0^1 (F(z))^{\sum_j m_j} dz \right] - cm_i.
\end{aligned}$$

Thus, incentives in the specified contest are strategically equivalent to those in an innovation tournament. Note, however, that since the strategy space in the innovation tournament is discrete, the exact isomorphism holds only for contests in which the strategy space is discrete.

Finally, using Leibniz's rule,

$$\begin{aligned}
\frac{\partial v(x_1, x_2, \dots, x_n)}{\partial x_i} &= \frac{\partial v(x_1, x_2, \dots, x_n)}{\partial (\sum_{j \neq i} x_j)} = \frac{\partial v(x_1, x_2, \dots, x_n)}{\partial (\sum_j x_j)} \\
&= -\frac{1}{c} \int_0^1 (F(z))^{\sum_j x_j} \ln F(z) dz > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 v(x_1, x_2, \dots, x_n)}{\partial x_i^2} &= \frac{\partial^2 v(x_1, x_2, \dots, x_n)}{\partial (\sum_{j \neq i} x_j)^2} = \frac{\partial^2 v(x_1, x_2, \dots, x_n)}{\partial (\sum_j x_j)^2} \\
&= -\frac{1}{c} \left(\int_0^1 \frac{\partial}{\partial x_i} \left((F(z))^{\sum_j x_j} \ln F(z) \right) dz \right) \\
&= -\frac{1}{c} \left(\int_0^1 (F(z))^{\sum_j x_j} \ln^2 F(z) dz \right) < 0
\end{aligned}$$

■

To illustrate this theorem, suppose $n = 2$ and ideas are uniformly distributed on $[0, 1]$. Then an innovation game $\gamma \in I(F, c, n)$ is strategically equivalent to a contest

$\Gamma \in C(v, p, n)$ with success function

$$p_i(x_1, x_2) = \frac{x_i}{x_1 + x_2}$$

and prize function

$$v(x_1, x_2) = \frac{1}{c} \left(\frac{x_1 + x_2}{x_1 + x_2 + 1} \right).$$

More generally, Theorem 1 reveals that one may view an innovation tournament as a standard contest in which the underlying “prize” is not only an increasing function of each firm’s own R&D effort, but also an increasing function of rivals’ efforts and aggregate efforts. Thus, apart from the well-known negative externalities on the probability of winning a contest, innovation tournaments also exhibit positive externalities among players’ efforts. The intuition is that in order to win the tournament a firm must surpass the rivals’ R&D efforts. Since higher R&D effort increases the likelihood of making a more valuable discovery, and since the value of the discovery determines the value of the prize, rivals’ R&D efforts have a positive “leap-frogging effect” on the value of the prize.⁶

Our next theorem establishes a relationship between innovation games and Tullock contests.

Theorem 2 *A Tullock game $\Gamma \in T(v, R, n)$ with $R = 1$ and discrete strategy space is strategically equivalent to an innovation game $\gamma \in I(F, c, n)$ in which the distribution*

⁶The R&D literature has identified another form of positive external economies. If knowledge of the R&D results leaks to other firms, then R&D activity gives rise to so-called “spillover effects” (Arrow, 1962).

of ideas is

$$F(y) = (G(y))^{\frac{1}{\sum_j m_j}}$$

and innovation costs are

$$c = \frac{1}{v} \left[1 - \int_0^1 G(z) dz \right].$$

Proof. As in the proof to Theorem 1, the payoff function for an arbitrary innovation game $\gamma \in I(F, c, n)$ may be written as

$$\pi_i(m_1, m_2, \dots, m_n) = \frac{m_i}{\sum_j m_j} \left[1 - \int_0^1 (F(z))^{\sum_j m_j} dz \right] - cm_i.$$

When

$$F(z) = (G(z))^{\frac{1}{\sum_j m_j}}$$

and

$$c = \frac{1}{v} \left[1 - \int_0^1 G(z) dz \right],$$

payoffs for the innovation game may be written as

$$\begin{aligned} \pi_i(m_1, m_2, \dots, m_n) &= \frac{m_i}{\sum_j m_j} \left[1 - \int_0^1 G(z) dz \right] - cm_i \\ &= c \left[\frac{m_i}{\sum_j m_j} v - m_i \right] \end{aligned}$$

Thus, incentives in the specified innovation game are strategically equivalent to those in a Tullock game with $R = 1$. Since the strategy space in the innovation game is discrete, the exact isomorphism holds only for contests in which the strategy space is discrete. ■

The proof of Theorem 2 reveals that a duality between innovation tournaments and Tullock games can be established in the presence of diseconomies of density in the total number of R&D experiments. This occurs when the profitability of a single R&D experiment declines with the total number of experiments in the industry. When these diseconomies of density exactly offset the leap-frogging effect identified in Theorem 1, an innovation tournament may be viewed as a Tullock game with $R = 1$.⁷

Next, we turn to patent races. It is easy to see that, in the limit as the interest rate tends to zero, a patent race $\gamma' \in P(h, v, r, n)$ with hazard rate $h(x_i) = x_i^R$, is formally equivalent to a Tullock game $\Gamma' \in T(v, R, n)$. In particular, note that player i 's payoff in a patent race can be expressed as

$$\pi_i(x_1, x_2, \dots, x_n) = v \frac{h(x_i)}{\sum_{j=1}^n h(x_j) + r} - x_i.$$

When $h(x_i) = x_i^R$ and the interest rate tends to zero, these payoff functions converge to those in the Tullock game. However, results for the Tullock game cannot be directly applied without imposing further restrictions. The reason is that the patent race game assumes $h(x_i)$ to be an increasing and concave function, defined on $[0, 1]$. For the duality to be exact, one must assume $R \leq 1$ (to obtain concavity). Furthermore, for $R \leq 1$, it is well-known from the literature on Tullock games that there exists a

⁷Substantial diseconomies of density appear to be present, for example, in industries where firms use patents simply to “colonize” unexplored areas of technology, such as the computer industry. See *The Economist*, “Patent Wars,” April 6, 2000.

unique symmetric Nash equilibrium in which, for all i :

$$x_i = \frac{(n-1)}{n^2}vR.$$

Hence, imposing an upper bound on v is sufficient to guarantee that $0 \leq h \leq 1$.

These observations, coupled with arguments similar to those in the proof of Theorem 2, yield the following duality results.

Theorem 3 *In the limit as r approaches 0, a patent race $\gamma \in P(h, v, r, n)$ with hazard rate*

$$h(x_i) = x_i^R, \text{ for } R \leq 1,$$

is strategically equivalent to a Tullock game $\Gamma \in T(v, R, n)$ with $R \leq 1$ and prize $v \leq n^2/(n-1)$. Furthermore, a patent race with hazard rate

$$h(x_i) = x_i$$

is, in the limit, strategically equivalent to an innovation tournament $\Gamma \in I(F, c, n)$ in which the distribution of ideas is

$$F(y) = (G(y))^{\frac{1}{\Sigma_j m_j}}. \tag{2}$$

Note that a decrease in the interest rate, r , is equivalent to shrinking the units in which time is measured. Hence, as r approaches zero, a patent race converges to its limiting static form. Specifically, when the cumulative distribution of valuable ideas is given by (2), a dynamic patent race converges to a static innovation tournament.

4 Applications and Concluding Remarks

We conclude by illustrating the practical utility of the above results. First, consider the rent-seeking literature. Chung (1996) analyzes a contest with success function

$$p_i(x_1, x_2, \dots, x_n) = \frac{x_i}{\sum_j x_j},$$

and compares the aggregate equilibrium effort with the socially efficient effort. He shows that rent-seeking contests generate socially wasteful (excessive) effort, provided the prize function is increasing in aggregate effort and satisfies a few additional assumptions. From Theorem 1, we know that an innovation tournament $\gamma \in I(F, c, n)$ is strategically equivalent to a contest with Chung's success function and in which the prize function is increasing in total effort. Furthermore, it is straightforward to verify that the associated prize function satisfies all of Chung's conditions. Thus, we may apply Chung's results from the rent-seeking literature to conclude that, in *any* innovation tournament $\gamma \in I(F, c, n)$, R&D investment is excessive relative to the social optimum.

Next, consider the R&D literature. Our results formally link the Fullerton and McAfee (1999) research tournament model (where n contestants compete to discover the best innovation in order to win a fixed prize, P) and innovation tournaments with endogenous prizes, $I(F, c, n)$. In particular, equation (1) in Fullerton and McAfee⁸ implies that their research tournament model is strategically equivalent to a Tul-

⁸Note that z_i in Fullerton and McAfee (1999) corresponds to m_i in our model.

lock contest $T(P, 1, n)$. Hence, it follows from our Theorem 2 that there exists an endogenous-prize tournament $\gamma \in I(F, c, n)$ that is strategically equivalent to the exogenous-prize research tournament model of Fullerton and McAfee.

In a similar fashion, one may use our theorems in conjunction with results in the contest literature to derive implications for innovation games. For instance, when the distribution of ideas satisfies the conditions under which an innovation tournament is strategically equivalent to a Tullock game with $R = 1$ (Theorem 2), or when the interest rate is small and a patent race is strategically equivalent to the Tullock game (Theorem 3), one may use results from the contest literature to establish that the Nash equilibrium of an innovation game is invariant to the sequence of moves (cf. Dixit, 1987; Baye and Shin, 1999) or leads to over investment in R&D (Dixit, 1987; Baik and Shogren, 1992).

Finally, our theorems permit one to use results in the patent literature to shed light on contests, innovation, and rent-seeking games. For example, Jensen and Showalter (2001) recently examined the impact of leverage on patent races, and found that debt acts as a commitment to lower R&D investments. Using the isomorphisms identified above, Jensen and Showalter's results have obvious implications for the impact of borrowing on the level of effort in rent-seeking and other contests.

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